Protected Import in Modular VDM: Towards a Coadaptation of Closed and Open Views of Modules in a Structured Specification

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Abstract. The complexities and the dynamics of evolving software development today require more than ever the provision of reusable building blocks and structuring methods in order to build larger and more complex specifications. This is the first in a series of papers towards a compositional semantics for modular structured VDM specifications. We provide a compositional extension of the denotational semantics for the flat VDM-SL, emphasising on *contextual structuring*. In addition, we discuss some non-interference and compositionality assumptions that underlie the structuring mechanisms of modular VDM specifications and introduce a new structuring assembly called *protected import* in order to control information flow in contextual structuring.

1 Introduction

VDM's origins lie in the definition of programming language semantics in the 1970s, but it has for many years been used in systems specification and development generally [18]. Areas to which VDM has been applied include railway interlocking systems [17], ammunition control systems [25], semantics of data flow diagrams [21], message authentication algorithms [26], relational database systems [31, 16] and medical information systems [6].[1] However, the ISO standard VDM-SL definition [22] does not provide a standard notion of a module syntax or its semantics, although some interesting approaches to defining modular VDM specifications are outlined in its appendices. In the absence of a standardised notion of a module, the modular construct introduced by IFAD [14] is now a *de facto* industrial standard.

In [11] we discussed a compositional (denotational) semantics for modular VDM specifications using a syntax that extends IFAD's modular construct.[2] In the present paper we emphasise on one of the most important contributions of that compositional semantics which is the coadaptation of closed and open views

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[1] A directory of VDM usage examples is available at http://www.ifad.dk/examples/examples.html.

[2] This was presented in the 2^{nd} *VDM Workshop* in York, September 2000 (in conjunction with ZB2000).

of modules in a structured VDM specification. A structured VDM specification is viewed as a (finite) collection of interrelated VDM modules. The denotational semantics we discussed in [11], and have used in this paper, is *compositional* in the following sense:

1. models of a structured specification are constructed using the models of the constituent modules as building blocks, in a way that reflects the structuring of the specification;
2. the semantics of a structured specification are derived from the semantics of the constituent modules with respect to certain constraints which are imposed by the specified interconnections between the constituent modules.

Consequently, the calculation of the semantics of a structured specification *decomposes* to the calculation of semantics for the constituent modules with respect to the *constraints* that are imposed by the explicitly specified module interconnections.

In general, one can distinguish the following basic types of modular structuring in state-oriented specification.

Contextual structuring which is mainly concerned with building the static part of a specification (non-state types and auxiliary functions)which provides the context for defining state and operations.

State structuring which focuses on the modular presentation of operations and state. Using this type of structuring one may capture state sharing, synchronisation and state dependency, therefore producing specifications that reflect the architectural structure of the implementation.

Dynamic state structuring which considers modules that represent(abstract) classes and allows dynamic state generation.

Of course this distinction is made for presentation and methodological purposes; one should expect a mixture of the above in most "real-life" applications.

This paper is one of the first part of a series of papers towards a *compositional* formal semantics for structured VDM specifications, and considers *contextual structuring*. A detailed account of *state structuring* and *dynamic state generation* will be provided in forthcoming papers. In contrast to what is often thought, contextual structuring does *not* simply serve presentation purposes. As we illustrate even at this level of structuring there are interesting interference and compositionality issues to be considered. This is a good reason for distinguishing *contextual structuring* as the obvious starting point.

The semantics discussed in this paper (and in [11]) are *generic* in the sense that they rely on weak assumptions placed upon the denotational semantics of the flat VDM language and will work with alternative denotational semantics in addition to that described in the ISO standard. We focus on the following features of this semantics.

1. The use of *dependent functions* as a mathematical basis for defining denotation assignment in structured specifications. This allows environments for the flat language to be used as building blocks for defining environments

for structured specifications. Hence, facilitating a transparent relationship between the models of flat and structured specifications.

2. The use of simple *"binding equations"* in order to capture the sharing of constructs via the export/import interface. This simplifies the semantic analysis of structured specifications
3. The distinction of *protected* import from the common (unprotected) import. If the import of a construct is declared to be protected, then the corresponding semantics ensure that the importing module does not interfere in the semantics of the imported construct. That is, *no emerging properties* from the importing to the source module are allowed. In order to be effective, protected import relies on some architectural assumptions which ensure that emerging properties cannot flow through some indirect information channel.

An outline and informal description of the basic structuring mechanisms considered in this paper is provided in section 2. The detailed definitions of these structuring mechanisms are provided in sections 5–7 following a summary of the required elements from the semantics of the flat language. For presentation purposes, the structured language semantics is introduced incrementally as is sketched in section 4. In section 8, we relate our approach with previous attempts and other related work. We close this paper in section 8 by summarising the possible continuation of this work and emphasising some outstanding issues. As far as this is possible, we follow the IFAD VDM-SL Tools syntax for modular structuring, provided in [14]. Whenever there is a divergence from the IFAD syntax, this will be highlighted.

2 An Outline of the Basic Structuring Mechanisms

We discuss the (denotational) semantics of the following clauses:

Export The export interface selects those constructs of the module specification which are *"public"*, ie., visible to the outside. Export aims to capture some aspects of information hiding, which is a pragmatic and somewhat informal concept but also important concept that *has* to be supported by a formal semantics . If such a formal notion is not provided then one should expect, for example, that formal support for refinement will not allow *not* modelling what is hidden.

Import The import interface *binds* modules in a whole: it allows a part of the visible constructs of an *"imported"* module to be (re)used as a building block for another *"importing"* module. As a result, import may allow information to flow in either direction: the definition of the imported construct, which is provided by the *host* module, enriches the specification of the *importing* module which may, in turn, impose *emerging properties* that further constrain the semantics of the imported construct. Hence, in the most general case an import may affect the semantics of both the importing and the host modules.

Different kinds of import give rise to diverse associations between modules. More particularly, we distinguish between the *protected* and *unprotected* import of a construct. In the former case, the import *conserves* the semantics of the imported construct as in the host module; no emerging properties affecting the semantics of this construct are allowed. In the latter case, the only purpose of the import is binding: the importing module may implicitly restrict the semantic values of the imported construct.

3 Elements of the flat language semantics

As a first step towards building a semantics of structured specifications, we identify basic operations in the semantics of the flat language. These are in turn.

Flat language models: The semantics of a flat language specification *doc* is given by the associated set of models. The semantic function **SemSpec**: $DOC \rightarrow \wp(ENV)$ such that

$$\mathbf{SemSpec}(doc) = \{env \in ENV \,|\, \mathbf{IsAModelOf}(env, doc)\}$$

assigns to each document *doc* a set of models selected from a collection ENV of "candidate" models, which are called environments. ENV contains all maps from identifiers to possible denotations for VDM constructs, and **IsAModelOf** is a predicate which checks whether a given environment satisfies the specification.

Satisfaction: Satisfaction is captured in the standardised flat language semantics by means of the predicate $\mathbf{IsAModelOf}(env, doc)$ which checks if a given environment *env* satisfies (in the formal sense) a given specification *doc*. Satisfaction can be further refined by introducing a relation $\mathbf{IsAModelOf}(env, frm)$ which checks if a given environment satisfies a formula frm, where frm consists of identifiers in $domain(env)$.

Language restriction: We facilitate the formal definition of information hiding by introducing a language restriction operator over the flat language. We introduce the obvious language restriction operator:

> *Let ID denote the set of construct identifiers that are defined in a specification doc and generate a language L. If $VID \subseteq ID$ generates a sublanguage VL of L, denoted by $VL \subseteq L$, then VL is the restriction of L to VID.*

Note that VID should be sufficiently rich to generate a set of well-formed formulae. The (language generated by the) construct identifiers $ID(doc)$ cannot be restricted to any arbitrary subset. Also note that, in the more general case, the identifiers that appear in a module M include those imported from other modules.

Renaming: Renaming provides the means for avoiding name clashes. It defines a bijective correspondence between existing and new names (identifiers) of constructs without affecting their semantics. Given a renaming $r{:}ID1 \rightarrow ID2$ and a set *envset* of environments for $ID1$ the semantic function **SemRename** produces an isomorphic set of environments for $ID2$ as follows:

$$\mathbf{SemRename}(r, envset) = \{env2 \mid env2 = r \circ env1, env1 \in envset\}$$

Model reduction: We use the obvious model reduction operation which is seen as the precomposition of a model with a language inclusion (possibly after renaming):

Let ID denote the set of construct identifiers that appear in a specification doc and let inc:$VID \subseteq ID$ be a language restriction. The reduct of an environment env for ID to an environment env' for VID is given by restricting the domain of env from ID to VID, hence by precomposing env with the inclusion of inc:$VID \to ID$, i.e. $env' = env \circ inc$.

Given a set envset of ID-environments and a language inclusion inc:$VID \to ID$ the semantic function **reduce** *produces the set of their VID-reducts:*

$$\mathbf{reduce}(VID, envset) = \{env \circ inc \mid env \in envset\}$$

Clearly, model reduction conserves all the *public* properties: an environment *env* for ID satisfies a *public* (VID-)property *exp* if and only if $env \circ inc$ satisfies *exp*.

Model expansion: Let inc:$VID \subseteq ID$ as above, and let $env1$, $env2$ be environments for VID and ID respectively. We call $env2$ a model expansion of $env1$ to the language of ID iff $\{env1\} = \mathbf{reduce}(VID, \{env2\})$. Given a set of environments for VID we write $\mathbf{expand}(ID, envset)$ to denote the set of model expansions of the environments in *envset*. That is,

$$\mathbf{expand}(ID, envset) = \{env \mid env \circ inc \in envset\}$$

If *env* is an environment for VID then the writing $\mathbf{expand}(ID, \{env\})$ or its paraphrase $\mathbf{expand}(ID, env)$ will denote the set of model expansions of *env* to ID.

4 An Overview of the Semantic Functions Definitions

A module specification can be divided into two parts: *the interface* (i.e., the exports, imports and parameters clauses) and the *body*. In general, a denotational model for a structured specification SP amounts to a function that assigns a flat language model of the (visible part of the) body of SP to the identifier of each module in the specification . Hence, if MID is the set of identifiers of the modules that constitute SP then

$$\mathbf{Models}(SP) \subseteq \mathbb{FUN}(MID, ENV_{mid})$$

Note that these models are **dependent functions**. That is, ENV_{mid} depends on the domain value $mid \in MID$. In particular the mappings in ENV_{mid} assign values to the (visible) construct identifiers in VID_{mid}, i.e. the identifiers of the constructs in the (exported) language of the module.

We introduce the semantics of a structured specification in the following steps:

1. In section 5 we introduce the semantics associated with the *export interface*.
2. We give a definition of models for specifications with *non-parameterised* modules in section 6, in the following steps:
   (a) we discuss the semantics of module binding via (unprotected) import,
   (b) we distinguish the protected import and justify the purpose of the new '`imports protected`' clause,
   (c) we introduce a method for deriving semantics for the *body* of a structured specification (built from non-parameterised modules) and
   (d) finally we extend this method by incorporating the hiding described by the export interface.

5 The Semantics of Exports

In this section we discuss the semantics associated to the export interface, in two steps.

1. In subsection 5.1, we consider a simple structured specification consisting of a single module that exports all and imports nothing.
2. In subsection 5.2, we generalise to the case that selected constructs are exported.

Because of space limitations, in this paper we focus on structured specifications consisting non-parameterised modules. The extension of this to the case of structured specifications with parameterised modules is summarised in [11].

5.1 A Simple Module

Let us first consider a structured specification SP consisting of a single module M with no imports and no parameters and no hiding (ie., where `exports all` is the only interface). A model of SP is a function that assigns a flat language model of the body of M to the module identifier for M. Hence, $\mathbf{SemStrucSpec}(SP) =$

$$\{m{:}\{id(M)\} \to ENV_{id(M)} \mid m(id(M)) \in \mathbf{SemSpec}(body(M))\}$$

where:

- $id(M)$ is the module identifier of M,
- $ENV_{id(M)}$ is the class of environments for the (visible) language of M, which in this case is the same as the flat language of $body(M)$, and
- $\mathbf{SemSpec}(body(M))$ is the set of models for the flat language specification $body(M)$ as defined in section 3.

Note that in the above definition every construct identifier is directly associated with its module identifier. In this way, name clashes between different modules are avoided. For simplicity, when the module M is fixed or its identifier is clear from the context we may refer to the construct c in M directly through $id(c)$.

Example 1. Assume that a structured specification consists of the following module

```
module MODULE1
    exports all;
    definitions
        functions
            f(x:nat) fx:nat
            pre TRUE
            post fx < 10;

            h(x:nat) hx:nat
            pre TRUE
            post (5 < hx) & (hx < f(x));
end MODULE1
```

The implicit definition of h affects the definition of f by forcing all models of MODULE1 to be functions with image values natural numbers between 7 and 9. This is emerging property has not been stated explicitly in the definition of function f. An example of an assignment constituting a model for this document is MODULE1 $\longmapsto$(f$\longmapsto \lambda v.9$, h$\longmapsto \lambda v.6$).

5.2 The Semantics of Exports

To provide semantics for information hiding which underpins the export interface and, to some extend, module binding we will use the notion of model reduction introduced in section 3.

Definition 1. *Let SP denote a structured specification consisting of a simple module M whose interface has no imports and no parameters, and such that M exports a collection VID of* public *constructs. Then:*
SemStrucSpec(SP) =
$\{m{:}\{id(M)\} \rightarrow ENV_{id(M)} \mid m(id(M)) \in \mathbf{SemHide}(VID, \mathbf{SemSpec}(body(M)))\}$
where the function **SemHide** *provides semantics for information hiding as defined in Definition 2.*

One way to provide semantics to information hiding is by identifying the set of models after the hiding with those models that agree with the reductions of models of the body of the module to the visible language.

Definition 2. *Let envset be a set of VID-environments and inc:$VID \subseteq ID$ be a language inclusion. The auxiliary semantic function* **SemHide** *is defined as:*

$$\mathbf{SemHide}(VID, envset) = \{mb \circ inc \mid mb \in envset\}$$

Consequently,

$$\mathbf{SemHide}(VID, \mathbf{SemSpec}(body(M))) = \mathbf{reduce}(VID, \mathbf{SemSpec}(body(M)))$$

Example 2. Consider the structured specification:

```
module MODULE2
    exports
        functions ff:nat->nat;
    definitions
        functions
            ff(x:nat) ffx:nat
            pre TRUE
            post ffx < 10;

            hh(x:nat) hhx:nat
            pre TRUE
            post (5 < hhx) & (hhx < ff(x));
end MODULE2
```

The models of this specification will provide semantic values for ff only. However, hh is used in order to calculate these values and then abstracted away. In effect, the above specification has the same semantics as the following version

```
module MODULE2
    exports
        functions ff:nat->nat;
    definitions
        functions
            ff(x:nat) ffx:nat
            pre TRUE
            post (6 < ffx) & (ffx < 10);
end MODULE2
```

5.3 Remarks

1. We note that, since a "no-junk" semantics is followed (i.e., environments are mappings with domain the visible language of the module), the set of models essentially amounts to the class of reducts of the (flat language) models of the body. Had a "junk-admissible" semantics approach (e.g. [2]) been followed in representing the environments assigned to module identifiers (i.e., these environments were mappings with domain all construct identifiers that appear in the document) then the following modification would apply: $\mathbf{SemHide}(idset, envset) = \{mv \in ENV | mv(id) = mb(id)$ for some $mb \in envset$ and every construct identifier id in $idset\}$.
2. Even with the presence of "junk" in the models, there is a significant difference between the above definition and the one proposed in [2]. In our case export does not simply hide the assignments to the non-visible symbols. These assignments are "forgotten" in the sense that they do not impose any direct constraint on the models of the module. Any model that agrees on the assignment of the visible part with a model of the module's body is accepted

as a model for the module. Hence, refinement that allows to avoid modelling what is hidden is supported.

6 Module Binding

In this section we define a semantic function for binding the bodies of the structured specification and then extend the definition to incorporate the *exports* interface. The semantic function for the bodies is defined in two steps: In subsection 6.1 we define a semantic function for binding without protection. In subsection 6.2 we extend this function to the case where some imports are required to be protected.

6.1 Unprotected Import

We provide a generic definition of a denotational semantics for the bodies of structured specifications (with unprotected import) which is rich enough to capture the information flow via the import of some constructs and the resulting mutual dependence between importing and imported modules.

Let SP be a structured specification consisting of a collection $M1 \ldots Mn$ of interrelated *non-parameterised* modules. Let Mi denote a module specification which imports a collection of constructs $IMPij$ from a module Mj For simplicity we assume that all construct identifiers in SP are distinct. (Otherwise some renaming may be needed.). Then, the following hold:

- $IMPij \subseteq VIDj \subseteq IDj$ where $VIDj$ is the collection of all constructs exported by Mj and IDj is the collection of all constructs that appear in the body of Mj,
- $IMPij \subseteq IDi$ where IDi is the collection of all constructs that appear in the body of Mi,
- $IMPij$ generates a sublanguage of the visible language of Mj and hence a sublanguage of $body(Mj)$,
- $IMPij$ generates a sublanguage of $body(Mi)$.

In the following, we define semantics for the body of the structured specification. The latter consists of the collection of interrelated bodies of the structured specification augmented with additional constraints induced by the structuring. These constraints take the form of *"binding equations"* which provide the semantic basis for capturing the dependence between the bodies of the modules. Then we localise the above semantics to each individual module of the structured specification.

Definition 3. *The set* **SemBodySpec***(SP) is a collection of MID-indexed families of environments which provide semantics for the bodies of the modules in structured specification. The set* **SemBodySpec***(SP) consists of these families of environments* $senv = (env_i)_{1 \leq i \leq n}$ *which satisfy the following conditions:*

1. $env_i \in$ **SemSpec**$(body(Mi))$ *for each* i *where* $1 \leq i \leq n$, *and*

2. $env_i(c) = env_j(c)$ *for each identifier* c *in* $IMPij$, $1 \leq i, j \leq n$;

where $(Mi)_{1 \leq i \leq n}$ *is an enumeration of the modules in* SP *and* env_i *is a shorthand for* $senv(id(Mi))$.

Definition 4. *By* **SemBodySpec**(i, SP) *we denote the set of the* i*-th projections of the families in* **SemBodySpec**(SP). *That is,*

$$\textbf{SemBodySpec}(id(Mi), SP) = \{mb(id(Mi)) | mb \in \textbf{SemBodySpec}(SP)\}$$

According to this definition, a model mb for the structured body of SP amounts to an n-tuple $\langle mb(id(M1)), \ldots, mb(id(Mn))\rangle$ which has the following properties.

1. each $mb(id(Mi))$ is a model of the flat specification $body(Mi)$
2. if Mi imports respectively constructs with identifiers in $IMPi_1..IMPi_k$ from modules $M_1..M_k$ then
   (a) $mb(id(Mi))$ is a common expansion to IDi of respectively the $IMPi_1$-,..., $IMPi_k$-reducts of modules $mb(id(M_1)), \ldots, mb(id(M_k))$.
   (b) each of $mb(id(M_1))..mb(id(M_k))$ is respectively an expansion to $ID_1..ID_k$ of the $IMPi_1$-,...,$IMPi_k$-reduct of $mb(id(Mi))$

The above 2a refers to the enrichment of the body of the importing modules with the properties of the imported constructs whereas the above 2b refers to the enrichment of the bodies of the imported modules with the emerging properties. The two enrichments in above 2 mutually depend on each other. However, this mutual dependence is reduced to a set of simple constraints realised by means of *"binding equations"* between certain denotation assignments when the semantics of the whole structured specification are considered. Clearly, there may be models of $body(Mi)$ which do not participate in any model of SP. That is, the inclusion **SemBodySpec**$(id(Mi), SP) \subseteq$ **SemSpec**$(body(Mi))$ generally holds but may not be invertible because of the emerging properties that may be induced via the module aggregation. From an engineering perspective, to constitute aggregates of modules via unprotected import is useful for reusing specification text and analysing the overall specification of complex systems where information flow between between components needs to be facilitated. In this respect, (unprotected) import provide the basis for designing *"Open"* structuring assemblies.[3]

Example 3. Consider a structured specification consisting of `MODULE1` of Example 1 and the following `MODULE3`.

```
module MODULE3
    imports
        from MODULE1
            functions f:nat->nat;
    exports
        functions g:nat->nat
```

[3] The term "Open" refers to the "Open-Closed" duality in the design of specification and code modules. See the conclusion of this paper and [23] for a further discussion.

```
    definitions
        functions
            g(x:nat) gx:nat
            pre TRUE
            post (MODULE1'f(x) < gx) & (gx < 8);
end MODULE3
```

The (unprotected) import identifies the semantic values of the imported `f` in the models of `Module3` and those in the models of `Module1`. Hence, the models of `MODULE1` are further restricted to those where `f` is the constant function $\lambda v.7$ and `h` is the constant function $\lambda v.6$. This illustrates that (unprotected) import allows constructs in different modules to interfere in each other semantics as if they were introduced simultaneously in the same module. Such liberality supports mutual imports. As we elaborate in the sequel this is not the case with protected import.

6.2 The Semantics of Protected Import

We introduce an alternative form of import which ensures that the information channel established by importing a set of constructs $IMPij$ from a *host* module Mj into an *importing* module Mi does *not* allow any information to flow from the *importing* module Mi to the *host* module Mj. In formal terms, this means that the import conserves the properties of the imported constructs in $IMPij$ as specified in the the context of the *host* module Mj. Hence, the semantics of the imported construct is protected against any emerging properties imposed by the import. Although the engineering purpose may be somewhat different, this kind of protection is formally related to the assurance of no interference from the importing module to the host module. After all, one cannot guarantee that any implementation of a module can be available for blind use elsewhere when emerging properties can imposed to the context of the module through unprotected import can implicitly alter the semantics of this module. By protecting the import we provide the contextual basis on which further non-interference conditions can be introduced. From a logical perspective, protected import amounts to a property enrichment that is conservative on the (shared) sublanguage that is formed by the constructs in $IMPij$.

One way to define the semantic function for (the aggregate body) of a structured specification with protected import is by first collecting the environments that statisfy a structured (partial) specification which ignores the associations that amount to protected imports while anticipating the information flow through unprotected imports and, then, restore in a stepwise manner the disregarded associations checking at each step that the non-interference assumptions relevant to the protected import are satisfied.

Before defining the semantic function for (non-parameterised) structured specifications with import protection we need to introduce the following auxiliary concepts. We assume an enumeration $(id(Mi))_{1\leq i\leq n}$ of the module identifiers in a structured specification and we write

$IMPij$ to denote the set of construct identifiers that a module Mi imports from a module Mj without protection;
PR_IMPij to denote the construct identifiers that a module Mi imports from a module Mj with protection;
inc_i:$PR_IMPij \subseteq IDi$ to denote the inclusion of the (construct) identifiers of a protected import into the set of construct identifiers of $body(Mi)$;
inc_j:$PR_IMPij \subseteq IDj$ to denote the inclusion of the set of (construct) identifiers of a protected import into the set of construct identifiers of $body(Mj)$;
MID to denote the set $\{i | 1 \leq i \leq n\}$ of the module identifiers in SP.

The following definition extends the notion of a submodel to structured specifications.

Definition 5. *Let INC:$X \subseteq MID$ and $SENV$ be a set of structured environments. Then,*

$$\mathbf{reduce}(X, senv_set) = \{senv \circ INC \mid senv \in SENV\}$$

The boolean function **Connect** checks if there is a possibly indirect *unprotected* association between a pair of identified modules.

Definition 6. $\mathbf{Connect}(i, j) =$

$IMPij \cup IMPji \neq \varnothing$ `or`
$\exists x \in MID.\ IMPix \cup IMPxi \neq \varnothing$ `and` $\mathbf{Connect}(x, j)$.

Note that **Connect** is well-defined because the number of modules and associations in a structured specification is always finite. Also note that $\mathbf{Connect}(i, j)$ and $\mathbf{Connect}(j, i)$ always give the same value.

Definition 7.

1. **PrDependCore** *is the set of module identifiers that protectively import a construct.*
   $\mathbf{PrDependCore}(SP) = \{i \mid \exists\ x \in MID.\ PR_IMPix \neq \varnothing\}$
2. **PrDepend** *is the set of module identifiers that are connected to some module whose identifier belongs to* **PrDependCore**. *That is,*
   $\mathbf{PrDepend}(SP) = \{i \mid \exists x \in \mathbf{PrDependCore}(SP).\ \mathbf{Connect}(i, x)\}$

The function **PrIndieCore** gives the set of module identifiers that do not belong to **PrDepend**.

Definition 8. $\mathbf{PrIndieCore}(SP) = MID - \mathbf{PrDepend}(SP)$

Note that the set **PrIndieCore** contains the module identifiers of the larger subsystem in SP whose semantics are *not* affected by the presence of the specified protected imports.

The following condition **FlowControl** describes an "architectural assumption" upon which the appearance of import protection relies in order for the semantics to guarantee no information flow from the importing to the host module at the presence of a protected import.

Definition 9.

$$\textbf{FlowControl} \;=\; \forall i,j \in MID.\ \texttt{if}\ PR_IMPij \neq \varnothing\ \texttt{then}\ \textbf{NoFlow}(i,j)$$

Where **NoFlow** is a boolean function ensuring that there is no information flow from Mi to Mj via another (possibly indirect) association.

Definition 10. $\textbf{NoFlow}(i,j) =$

$(i \neq j)$ `and`
$(IMPij \cup IMPji \cup PR_IMPij = \varnothing)$ `and`
$\forall\ x \in MID.$
`if` $(i \neq x$ `and` $j \neq x$ `and` $IMPix \cup IMPxi \cup PR_IMPxi \neq \varnothing)$ `then` $\textbf{NoFlow}(x,j)$.

Note that **NoFlow** is well-defined because the number of modules and associations in a structured specification is always finite.

If $\textbf{NoFlow}(i,j)$ holds then

1. Mi cannot import any constructs from Mj without protection;
2. Mj cannot import any constructs from Mi at all – hence mutual imports are excluded at the presence of import protection;
3. if Mi is further associated with Mj via third parties then **NoFlow** is applied recursively until the modules that directly associate with Mj are singled out and the above basic two conditions apply.

Furthermore **FlowControl** implies $\textbf{PrIndieCore} \neq \varnothing$. (This implication is further elaborated in Remark 1 (subsection 6.3) and illustrated in Example 5.)

We are now ready to modify Definition 3 in order to incorporate protected imports. We write SP^- to denote the (partial) structured specification that is produced from SP by eliminating all protected imports and appropriately renaming the protectively imported constructs so that they are distinguished from the constructs in the host module.

Definition 11. *The set* **ProtectSemBodySpec***(SP) is a collection MID-indexed families of environments which provide semantics for the bodies of the modules in structured specification.*

For each i in MID, let $PRi = \{j \mid PR_IMPij \neq \varnothing\}$ and rd_i denote the inclusion $PRi \subseteq MID$. Recall that inc_i:$PR_IMPij \subseteq IDi$, inc_j:$PR_IMPij \subseteq IDj$ denote the inclusion of the (construct) identifiers of a protected import into the set of construct identifiers respectively of body(Mi) and of body(Mj).

The set **ProtectSemBodySpec***(SP)is calculated by the following procedure.*

If FlowControl then

1. *Initialise*
   - Pos_Models = $\textbf{SemBodySpec}(SP^-)$
   - $CHECKED = \textbf{PrIndieCore}$
2. *While $CHECKED \subset MID$ do*
   (a) *Initialise $INCREMENT = \varnothing$*
   (b) *For each $i \in MID$ such that $PRi \subseteq CHECKED$ do*

i. *Define* PRi$\textsc{Models}$ = **reduce**(PRi, Pos_Models)

ii. **If**

$\forall mr \in$ PRi$\textsc{Models}$ $\exists mb \in$ Pos_$\textsc{Models}$. $mb \circ rd_i = mr$

and $\forall\ j \in PRi\ .\ mb(i) \circ inc_i = mb(j) \circ inc_j$

then

Update Pos_$\textsc{Models}'$ = $\{mb \in$ Pos_$\textsc{Models}$ | $mb(i) \circ inc_i = mb(j) \circ inc_j$,

where$j \in PRi$ *and* $(mb(j))_{j \in PRi} \in PRi_MODELS\}$

else

Update Pos_$\textsc{Models}'$ = $\varnothing$.

iii. *Update* $INCREMENT' = INCREMENT \cup \{i\}$

(c) *Update* $CHECKED' = CHECKED \cup INCREMENT$

3. *Define* **ProtectSemBodySpec**(SP) = Pos_$\textsc{Models}$

else

Define **ProtectSemBodySpec**$(SP) = \varnothing$

The fundamental differences between this definition and Definition 3 are that, firstly, this definition is conditional on **FlowControl** and, secondly, we have to stepwisely interpolate the condition 2(b)ii to this definition. Condition 2(b)ii asserts that no potential model of Mi in SP is eliminated because of emerging properties that are imposed by Mi on PR_IMPij.

Example 4. Consider a structured specification consisting of `MODULE2` of Example 2 and the following `MODULE4`.

```
module MODULE4
    imports
        from MODULE2 protected
            functions ff:nat->nat
    exports
        functions k:nat->nat;
    definitions
        functions
            k(x:nat) kx:nat
            pre TRUE
            post (MODULE2'ff(x) < kx);
end MODULE4
```

Firstly, the **FlowControl** architectural assumption is checked. This reduces to the validation of **NoFlow**$(4, 2)$. Secondly, the set $CHECKED = \{$`MODULE2`$\}$ is formed. Thirdly, the set Pos_$\textsc{Models}$ is calculated. In doing this, the information that `ff` is imported from `MODULE2` is not considered. Hence, any semantic value of `ff` that is compatible with the definition of `k` is accepted: Any tuple $(\texttt{ff} \longmapsto \lambda v.F(v), \texttt{k} \longmapsto \lambda v.K(v))$ where $F_4(n) < K(n)$, for every natural number n, constitutes an accepted model for *body*(`MODULE4`). In analogy, any tuple $(\texttt{ff} \longmapsto \lambda v.F_2(v), \texttt{k} \longmapsto \lambda v.H(v))$ where $5 < H(n) < F_2(n) < 10$, for every natural number n, constitutes an accepted model for *body*(`MODULE2`). At this stage, any combination of the above assignment tuples constitutes a possible

Fig. 1. A pictorial overview of a structured specification SP with the coadaptation of protected and unprotected import, its corresponding partial specification SP^- where the protected associations are disregarded, and its subsystem **PrIndieCore**(SP) whose semantics are not affected by the import protection.

model of $body(SP)$. Fourthly , the expandability condition 2(b)ii is verified: Given some $6 < F(n) < 10$ one can find a possible model of SP such that $F_4(n) = F_2(n)$ and $F_2(n) = F(n)$. Finally, the models of $body(SP)$ are those assignments such that $5 < H(n) < F_2(n) < 10,\ F_2 = F_4,\ F_4(n) < K(n)$. (See also section 6.3.Remark 2.)

Example 5. Consider the extension of the structured specification presented in Example 4 with the following module.

```
module MODULEM
    imports
        from MODULE2 protected
            functions hh:nat->nat
        from MODULE4
            functions k:nat->nat
    definitions
        functions
            m(x:nat) mx:nat
            pre TRUE
            post (MODULE4'k(x) < mx) & (mx < 10+MODULE2'hh(x));
end MODULEM
```

By appending `MODULEM` to the structured specification of Example 4 we have produced a structured specification which is still satisfiable but selectively restricts the variety of model expansions of `ff` that are models of `MODULE4`. For example, the tuple $(\texttt{ff} \longmapsto \lambda v.8, \texttt{k} \longmapsto \lambda v.8 + v)$ is in the `MODULE4`- projection of some model of the structured specification of Example 4 which cannot participate in any model of the specification of this example. Had $\texttt{k} \longmapsto \lambda v.8 + v$ been accepted as a semantic value for `k`, the induced constraint that $10 + 8$ bounds `k` for all natural numbers would be violated. The `MODULE4`-projections of other

acceptable models such as ($\mathtt{ff} \longmapsto \lambda v.8, \mathtt{k} \longmapsto \lambda v.8 + (v \ mod \ 2)$) take care that $\mathtt{ff} \longmapsto \lambda v.8$ is maintained as a possible semantics for `ff`.

Note that this structured specification satisfies the **FlowControl** structuring scheme and that **PrIndieCore** = {`MODULE2`}, as is depicted in Figure 1. Furthermore, the information that for every $mb \in$ Pos_Models, $mb(id(\mathtt{MODULE4}))(id(\mathtt{k})) = mb(id(\mathtt{MODULEM}))(id(\mathtt{k}))$ has been accounted at the beginning of the procedure described in Definition 11 and is preserved in every further step.

Example 6. An alternative and equally feasible, version of this example would be to have `MODULEM` import `ff` protected from `MODULE2`. Another interesting example would be to have `MODULE4` import `f` protected from `MODULE1` and `MODULE3` import `f` from `MODULE1` (unprotected) as in Example 3. In this case `MODULE3` would interfere by imposing emerging properties on `MODULE1` and hence eliminating possible semantic values for `f` as in Example 3 and `MODULE4` would be required to provide expansions from the remaining semantic values of `f`. Such architectures are permitted by **FlowControl** and allow the designer to enrich the specification of the importing or host modules through interaction with other subsystems while guaranteeing that information does *not* flow from the importing to the host module.

6.3 Remarks

1. Notably **FlowControl** implies **PrIndieCore** $\neq \varnothing$. Indeed, **FlowControl** implies $\exists i.i \in$ **PrIndieCore** by disallowing cyclic imports at the presence of import protection, on the one hand, and by ensuring that $\forall i, j.(PR_IMPij \neq \varnothing \wedge j \notin$ **PrDependCore**$) \Rightarrow j \neq$ **PrDepend**, on the other hand.
   For example, if Mi imports from Mj with protection and there is some module Mx which is associated with Mi then the only association between Mx and Mj allowed is protected import from Mj into Mx. As we demonstrate in Example 5, this architectural condition disallows eliminating any acceptable semantic values for the protected constructs that could be caused by uncontrolled implicit information flow, on the one hand, and facilitates eliminating unfavourable expansions by allowing information flow to the importing module as far as this does not affect the semantics of the host module. This controlled enrichment of the importing in a way that does not affect the host module provides the formal basis for various practically useful and theoretically interesting sharing schemes.
2. The protection in importing from `MODULE2` into `MODULE4` in Example 4 ensures that there are no emerging properties from `MODULE4` on `ff`. Hence, `MODULE4` has to accommodate all possible semantic values for `ff` which are admitted by the specification of `MODULE2` when the import is not considered. Whereas there may be some semantic values for `k` which admitted by the specification of `MODULE4` when the import is not considered and are then eliminated when the *"binding equations"* are applied. This way, information may flow from the host to the importing module but not in the other direction.

3. Note that the import of `f` from `MODULE1` into `MODULE3` presented in Example 3 cannot be protected. Declaring this import as protected will introduce a contradiction and produce a structured specification satisfied by an empty class of models. This is because, semantic values such as $\lambda v.8$ or $\lambda v.(6 + v \ mod \ 2)$ which are accepted with respect to the function definitions in `MODULE1` are excluded by `MODULE3`.
4. The **FlowControl** architecture scheme forces contextual decomposition of a module M into independent segments, each of which can have all its (public) constructs (protectively) imported into M. This is because the **FlowControl** assumes that all the constructs encapsulated in a module depend on each other by default. Hence, if there are constructs c_1 and c_2 which are declared in M but their semantics are independent one is not allowed to have some other module M' import c_1 with protection and c_2 without. However, if one decomposes M into two independent (sub)modules $M1$ and $M2$ respectively hosting c_1 and c_2 then M' may import c_1 from $M1$ with protection and c_2 from $M2$ without protection. If one wishes to avoid forcing such further segmentation of M then **FlowControl** will have to be refined to deal with dependence of (imported) constructs instead of modules, therefore drastically increasing the complexity of analysing the structuring architecture.

6.4 The engineering value of import protection

In the general case, protected import is particularly useful for sharing code and providing *reference-only* access to shared data. In the case of *contextual structuring*, protected import is useful for for *sharing separately implemented*, system-wide types and basic functions which are naturally associated with these types. The specification of such types can be provided in a *stateless* module SWT. By protecting the import of types and functions from SWT one ensures that a single copy of code will be present in the final product and that the correctness of this code will depend on the specification of SWT only. A particular case of this is to specify abstract (mathematical) data types specified in an operationless shared module and import their constructs in any other machine. Stateless and operationless modules may not need implementing; they provide a library of useful mathematical concepts that ease the specification of algorithms and architectures, and are usually "programmed away" during the development. From that perspective, the use of protected import from this modules is analogous to the use of SEES in B. (See [10] for further details on this matter.)

7 The Semantics of Structured Specifications

The overall semantic function is defined by abstracting away what is not public.

Definition 12. *The semantic function for a structured specification (which does not contain any parameterised modules) is defined as follows.*
SemStrucSpec$(SP) =$

$\{m{:}MID \to ENV_mid \mid$
$m(id(Mi)) \in \mathbf{SemHide}(VIDi, \mathbf{ProtectSemBodySpec}(id(Mi), SP))$
for each $1 \leq i \leq n\}$.

Notably, the above semantics is "sensible" (as opposed to "agnostic") to the export clauses that may occur in the modules of SP. For example, if SP contains just two modules $M1$ and $M2$ such that $M2$ imports $IMP1$ from $M1$ then SP will be given different semantics if $M1$ exports all compared to, say, when $M1$ exports $IMP1$ only. The reason is that the export describes the part of a module that is visible to the environment and therefore imposes implementation constraints on the specification. This distinction is important if a notion of structured refinement is considered in the future because all visible part of each a module may need to be refined in parallel.

Furthermore as we elaborated in the previous section, this definition of structured specifications accepts mutual *unprotected* imports. In this case, the only constraint is that, in each structured model, all imported constructs will be assigned to identical values in the imported and the (body of the) importing modules. Hence, the problem of calculating these values is moved to the flat language semantics where it is treated with respect to the already standardised methods. It is, however, necessary to prohibit mutual dependencies at the presence of a protected import.

8 Conclusion and Further Work

In this paper we discussed *compositional* denotational semantics for structured VDM specifications emphasising on the *contextual structuring*. In contrast to what is often thought, there are interesting and non-trivial interference and compositionality issues to be considered even at this level of structuring. The semantics discussed in this paper are *generic* in the sense that they rely on very weak assumptions placed upon the denotational semantics of the flat VDM language and will work with a variety of alternative denotational semantics in addition to the semantics described in the ISO standard. An overview of this semantics which also covers structured specifications with parameterised modules and parameter instantiation is provided in [11].

In this paper we focused on the distinction of a *protected* import from the common (unprotected) import. We provided semantic functions to ensure that, when the import of a construct is declared to be protected, the importing module does not interfere in the semantics of the imported construct. This means that the information flow channel that is opened via a protected import does not allow any information to flow from the importing module to the module that hosts the imported constructs. In other words, a protected import does not allow any *emerging properties* from the importing module. Import protection has been associated with an architectural condition upon which it relies to disallow any information flow via indirect associations between modules.

At least in formal terms, there is an analogy between the distinction of *protected* versus *unprotected* import and the *open – closed* duality principle (e.g.

Meyer [23]). Here "open" means building larger systems by extensions, e.g. when appending or amalgamating specification modules. "Closed" means building an encapsulated component available for blind use elsewhere, e.g. when linking independently constructed code modules. Protected imports underlie the specification of "closed" structuring assemblies which provide the means for building an encapsulated component available for blind use elsewhere as when linking independently constructed code modules. The, usual, (unprotected) imports underlie "open" structuring assemblies which are useful for building larger systems by extensions, e.g. when appending or amalgamating specification modules. Although protected imports may not capture all aspects of a "closed" structuring assembly, "closed" structuring assemblies *rely* on the assumption that there are no emerging properties to the context of the associated modules in order to ensure that the resulting encapsulated component can be available for blind use elsewhere: One cannot guarantee that any implementation of a module can be available for blind use elsewhere if emerging properties imposed through unprotected import can implicitly alter the semantics of this module.

A relevant, elaborate analysis of the non-interference and compositionality assumptions that underlie contextual structuring mechanisms of the B-Method is presented in [10]. In that paper we show how such assumptions can be violated by inducing emerging properties and therefore altering the *static context*. To avoid such violation, a set of contextual proof obligations related to the static part of the specification have to be considered. In [10] we provide a set of proof obligations which are associated with structuring the *static context* of an abstract machine, a refinement or an implementation in B, that is necessary and sufficient to ensure that the properties of the (static) *context* of the imported component are *conserved*, i.e. that they are preserved but not enriched. In the present paper we highlight analogous constraints on the VDM Modules structuring mechanisms at the denotational level. In future work we will provide an matching extension of axiomatic semantics for VDM [15, 3] based on proof obligations which facilitate proof decomposition by capture the structuring assemblies which are defined in this paper by denotational means.

Other contributions presented in this paper include

1. The use dependent functions to facilitate a transparent relationship between the denotational models of flat and structured specifications.
2. The use of simple *"binding equations"* in order to capture the information flow via the export/import interface.

Although we have used denotational semantics as a means of introducing these ideas to the VDM community, our major goal is to derive proof obligations in order to verify these structuring assemblies and to facilitate proof decomposition in modular VDM specifications. It is through (semi)automated proof support, that the structuring mechanisms discussed in this paper can be best utilised. In the following paragraphs we summarise some other interesting continuations of the reported research.

The obvious continuation of this research is towards an extension of this semantics to capture state structuring. Although, in principle the mechanisms

discussed in this paper provide sufficient mathematical tools for capturing state structuring, further issues have to be considered. In recognition of the fact that state structuring is the most important aspect of modular VDM specifications, we believe that, in addition to being formally cogent, the treatment of state has to rely upon mechanisms that are friendly to the designer of the specification and facilitate an efficient engineering practice. This may involve presenting as "primitive" structuring assemblies some practically useful patterns which correspond to different combinations of hiding and protected or unprotected imports.

In order to make the most out of modular structuring the agreed notion of refinement needs to be extended so that the following architectural assumptions are satisfied.

1. a refinement between structured specifications in the modular language corresponds to a family of (parallel) module-to-module refinements which also preserves the structuring of modules;
2. compositionality of refinement shall also extend from the flat language to the module language: the composition of two subsequent refinements between structured specifications shall be derived from the composition of the subsequent module-to-module component refinements;
3. a refinement between structured specifications in the modular language flattens to a refinement between the corresponding flattened specifications.

The concepts of parallel refinement have been successfully applied in category-theory based software engineering environments such as Specware [28, 29]. A further study on modularity for diagrammatically structured specifications and parallel refinement currently on going [7, 8]. A rather restricted instance of a similar idea has been applied for implementations in B [1, 20].

Other interesting topics for further work include extending the existing interface of the VDM-Modules in order to capture *event-based interaction* (i.e., a module sending and receiving events to another module before, after and during operations), multiple export interfaces (as in Java and ActiveX), the dynamic creation of associations between modules via a request to a purpose-built "broker" component (as in CORBA) and self-adaptive "intelligent" interfaces which can be seen as generic imoprt/export interfaces that are instantiated on demand and may reconfigure at real-time. (See [27] for a further discussion and motivating on the added value of some of these extensions in software construction.)

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