PhD thesis proposal
”Pointfree Program Calculation - Theory and Applications”

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1 Summary

This project aims at developing and extending the well-known technique of encoding predicate logic and set-theory in the binary relation calculus, for generic program calculation purposes. The context of this work is the relational theory of datatypes (rather than a calculus of total functions) which can be framed into allegory theory [2], a generalization of category theory.

When software designers refer to the relational calculus, by default what is understood is the calculus of n-ary relations studied in logics and database theory, and not the calculus of binary relations which was initiated by De Morgan in the 1980s [4] and eventually became the core of the algebra of programming [3, 2, 1].

This thesis intends to show that the use of the pointfree binary relation calculus is beneficial in several respects. First, the fact that pointfree notation abstracts from ”points” or variables makes the reasoning more compact and effective. Second, proofs are performed by easy-to-follow calculation. Third, one is able to generalize the original theory.

As a first result of this research, a *pointfree* treatment of relations (their properties, their operators, and the laws that govern them) was captured in a type-directed strategic rewriting system for transformation of relational expressions. This rewriting tool can be used to simplify relational proof obligations and ultimately reduce them to tautologies. Is easy to show how such reductions provide extended static checking for design contraints commonly found in software modeling and development.

References

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