A Framework for *Point-free* Program Transformation

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IFL, 2005

Outline of Part I
Basic Theory

Outline of Part II
Tools and Libraries

Outline

What Is *Point-free*
In Functional Context

Two different programming styles

- In ***pointwise*** – Using variables
- In ***point-free*** – Without the use of variables

Point-free language

- categorically-inspired combinators
- algebraic data types as fixed points of functors
- polytypic recursion patterns

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Appropriate for equational reasoning
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difficult for programming in practice.

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The Framework

Main topics

- Automatic conversion of programs to *point-free*
- *Point-free* manipulation
- *Point-free* rule-driven simplification

Use of the *Haskell* programming language.

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Tools And Libraries

Developed tools and libraries

Pointless

- *Haskell* library for *point-free* programming
- type-checking and execution of *point-free*
- type-parameterized recursion patterns

- automatic conversion to *point-free*
- recursive functions to *hylomorphisms*
- intermediate data structure inference

- *point-free* code manipulation
- simplification of *point-free* code
- program transformation

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Pointless	- *Haskell* library for *point-free* programming - type-checking and execution of *point-free* - type-parameterized recursion patterns
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SimpliFree	- *point-free* code manipulation - simplification of *point-free* code - program transformation

Outline

Types
Definition

$$
\begin{array}{rll}
A, B ::= & 1 & - \text{ single element type} \\
| & A \to B & - \text{ functions} \\
| & A \times B & - \text{ cartesian product} \\
| & A + B & - \text{ separated sum} \\
| & \mu F & - \text{ recursive (regular) type,} \\
 & & \quad \text{defined as the fixed point of a functor}
\end{array}
$$

$$
\begin{array}{rll}
F, G ::= & \mathsf{Id} & - \text{ identity functor} \\
| & \underline{A} & - \text{ constant functor} \\
| & F \otimes G & - \text{ lifted product bifunctor} \\
| & F \oplus G & - \text{ lifted sum bifunctor} \\
| & F \circledcirc G & - \text{ composition of functors}
\end{array}
$$

Types
Examples

Booleans, Naturals and Lists

- $\mathsf{Bool} = 1 + 1$
- $\mathsf{Nat} = \mu(\underline{1} \oplus \mathsf{Id})$
- $\mathsf{List}\ A = \mu(\underline{1} \oplus (\underline{A} \otimes \mathsf{Id}))$

Pointwise Language

Syntax

$$
\begin{array}{rcl}
L, M, N & ::= & \star \mid x \mid M\ N \mid \lambda x.\ M \mid \\
& & \langle M, N \rangle \mid \mathsf{fst}\ M \mid \mathsf{snd}\ M \mid \\
& & \mathsf{case}\ L\ M\ N \mid \mathsf{inl}\ M \mid \mathsf{inr}\ M \mid \\
& & \mathsf{in}_{\mu F}\ M \mid \mathsf{out}_{\mu F}\ M \mid \mathsf{fix}\ M
\end{array}
$$

Examples

$$\mathsf{in}_{\mathsf{Nat}} : 1 + \mathsf{Nat} \to \mathsf{Nat} \qquad \mathsf{out}_{\mathsf{Nat}} : \mathsf{Nat} \to 1 + \mathsf{Nat}$$

$$
\begin{array}{rcl@{\qquad}rcl}
\mathsf{zero} & : & \mathsf{Nat} & \mathsf{succ} & : & \mathsf{Nat} \to \mathsf{Nat} \\
\mathsf{zero} & = & \mathsf{in}\ (\mathsf{inl}\ \star) & \mathsf{succ} & = & \lambda x.\ \mathsf{in}\ (\mathsf{inr}\ x)
\end{array}
$$

$$
\begin{array}{rcl}
\mathsf{swap} & : & A \times B \to B \times A \\
\mathsf{swap} & = & \lambda x.\ \langle \mathsf{snd}\ x, \mathsf{fst}\ x \rangle
\end{array}
$$

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\begin{array}{rcl}
\mathit{length} & : & \mathsf{List}\ A \to \mathsf{Nat} \\
\mathit{length} & = & \mathsf{fix}\ (\lambda f.\ \lambda x.\ \mathsf{case}\ (\mathsf{out}\ x)\ (\lambda y.\ \mathsf{zero}) \\
& & \qquad (\lambda y.\ \mathsf{succ}\ (f\ y)))
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Point-free Language
Syntax

Constants

fst snd inl inr in out id

$\text{bang} : A \to 1 \qquad \text{ap} : (A \to B) \times A \to B$

Combinators

$(\cdot \circ \cdot) : (B \to C) \to (A \to B) \to A \to C$

$(\cdot \vartriangle \cdot) : (A \to B) \to (A \to C) \to A \to (B \times C)$

$(\cdot \triangledown \cdot) : (A \to C) \to (B \to C) \to (A + B) \to C$

$\overline{\cdot} : (A \times B \to C) \to A \to B \to C$

Combinators for the product, sum and exponention functors on functions were also derived.

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Point-free Language
Implicit Recursion

Recursion through the *hylomorphism* recursion pattern.
Allows the definition of any fixed point.

Defined in *pointwise* as

$$\begin{array}{lcl} \text{hylo}_{\mu F} & : & (F\ B \to B) \to (A \to F\ A) \to A \to B \\ \text{hylo}_{\mu F} & = & \lambda g.\ \lambda h.\ \text{fix}(\lambda f.\ g \circ Ff \circ h) \end{array}$$

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Some *Point-free* Laws

cata-FUSION

$$f \circ (\!|g|\!)_{\mu F} = (\!|h|\!)_{\mu F} \quad \Leftarrow \quad f \circ g = h \circ F\ f \quad \text{cata-FUSION}$$

More laws

$$(f \times g) \circ (h \times i) = f \circ h \times g \circ i \qquad \times\text{-FUNCTOR}$$

$$(f \triangledown g) \circ (h + i) = f \circ h \triangledown g \circ i \qquad +\text{-ABSOR}$$

$$f \circ (g \triangledown h) = f \circ g \triangledown f \circ h \qquad +\text{-FUSION}$$

$$(f \times g) \circ swap = swap \circ (g \times f) \qquad \text{swap-NAT}$$

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Example: Reverse

Pointwise Definition

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reverse [] = []
reverse (x:xs) = cat (reverse xs, wrap x)
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$$reverse_t\ l\ y = \overline{cat}\ (reverse\ l)\ y$$

$$reverse_t = \overline{cat} \circ reverse$$

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$$reverse = (\!|\, nil \triangledown (cat \circ swap \circ (wrap \times \mathrm{id}))\,|\!)_{List\ A}$$

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Cata-FUSION for lists

$$f \circ (\!|g|\!)_{\mathrm{List}\ A} = (\!|h|\!)_{\mathrm{List}\ A} \Leftarrow f \circ g = h \circ$$

$$reverse_t = (\!|h|\!)_{\mathrm{List}\ A}$$
$$\overline{cat} \circ nil \triangledown (cat \circ swap \circ (wrap \times \mathrm{id})) = h \circ (id + id \times \overline{cat})$$

Cat associativity

$$\overline{cat} \circ cat = comp \circ (\overline{cat} \times \overline{cat}) \qquad \text{cat-ASSOC}$$

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= & \quad \{ +\text{-Fusion} \} \\
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= & \quad \{ \text{cat-Assoc} \} \\
 & \overline{cat} \circ nil \triangledown comp \circ \qquad\qquad\qquad \circ (wrap \times \mathrm{id}) \\
= & \quad \{ \text{swap-Nat} \} \\
 & \overline{cat} \circ nil \triangledown comp \circ swap \circ \\
= & \quad \{ \times\text{-Functor} \} \\
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 & \overline{cat} \circ nil \quad comp \circ swap \circ (\overline{cat} \circ wrap \times \mathrm{id}) \circ \\
= & \quad \{ +\text{-Absor} \} \\
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$$h \circ (\mathrm{id} + \mathrm{id} \times \overline{cat}) =$$

$$\begin{bmatrix}
 & \overline{cat} \circ (nil \triangledown cat \circ swap \circ (wrap \times \mathrm{id})) \\
= & \quad \{ +\text{-FUSION} \} \\
 & \overline{cat} \circ nil \triangledown \overline{cat} \circ cat \circ swap \circ (wrap \times \mathrm{id}) \\
= & \quad \{ \text{cat-ASSOC} \} \\
 & \overline{cat} \circ nil \triangledown comp \circ (\overline{cat} \times \overline{cat}) \circ swap \circ (wrap \times \mathrm{id}) \\
= & \quad \{ \text{swap-NAT} \} \\
 & \overline{cat} \circ nil \triangledown comp \circ swap \circ (\overline{cat} \times \overline{cat}) \circ (wrap \times \mathrm{id}) \\
= & \quad \{ \times\text{-FUNCTOR} \} \\
 & \overline{cat} \circ nil \triangledown comp \circ swap \circ (\overline{cat} \circ wrap \times \overline{cat}) \\
= & \quad \{ \times\text{-FUNCTOR} \} \\
 & \overline{cat} \circ nil \quad comp \circ swap \circ (\overline{cat} \circ wrap \times \mathrm{id}) \circ \\
= & \quad \{ +\text{-ABSOR} \} \\
 & (\overline{cat} \circ nil \triangledown comp \circ swap \circ (\overline{cat} \circ wrap \times \mathrm{id})) \circ \\
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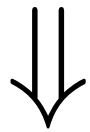
\end{array}
$$

Example: Reverse

Pointwise results

Original function

```
reverse [] = []
reverse (x:xs) = cat (reverse xs, wrap x)
```



Resulting function

```
reverse_t [] y = y
reverse_t (x:xs) y = reverse_t xs (x:y)
```

Outline

Basic Combinators

Allows the definition of *point-free* code in *Haskell*.

Point-free constants and non-recursive combinators directly translated to *Haskell*.

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Recursion
Functors

Based on generic programming library *PolyP*.
Data types viewed as fixed points of functors.

New type class with functional dependency

```
class (Functor f) => FunctorOf f d | d -> f
  where inn' :: f d -> d
        out' :: d -> f d
```

Regular functor combinators

```
newtype Id x          = Id {unId :: x}
newtype Const t x     = Const {unConst :: t}
data (g :+: h) x      = Inl (g x) | Inr (h x)
data (g :*: h) x      = g x :*: h x
newtype (g :@: h) x = Comp {unComp :: g (h x)}
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Implicit Coercion
Functor types representation

To avoid using functor combinators in functions.

Elements defined using functor combinators
$\Rightarrow$ Standard *Haskell* types

Functor evaluation class

```
class Rep fa rep | fa -> rep
  where to :: fa -> rep
        from :: rep -> fa
```

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Examples

Hylomorphism examples

```
fact :: Int -> Int
fact = hylo (_L :: [Int]) f g
  where g = (id -|- succ /\ id) . out
        f = one \/ mult


fold (_::d) g = hylo (_L::d) g out


length = fold (_L :: [a])
              (inn . (id -|- snd))
```


Outline

What is DrHylo

- Derives *point-free* definitions for a subset of *pointwise Haskell*;
- Resulting code can be executed using the Pointless library;
- 3 important steps:
  - Pattern matching removal;
  - Conversion of non-recursive definitions to *point-free*;
  - Explicit recursion removal.

Pattern Matching Removal

Towards *Haskell* code

- `FunctorOf` instances derived automatically;
- Constructors replaced by fixpoint definitions;
- Enough to pattern match over constructor `in`, sums, pairs and the constant $\star$;
- New constructor in the *pointwise* definition

$$\begin{array}{rcl} P & ::= & \star \mid x \mid \langle P, P \rangle \mid \mathsf{in}\ P \mid \mathsf{inl}\ P \mid \mathsf{inr}\ P \\ M, N & ::= & \ldots \mid \mathsf{match}\ M\ \mathsf{with}\ \{P \to N;\ \ldots; P \to N\} \end{array}$$

- Each *match* is converted to the core λ-calculus.

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Pointwise to *Point-free*
Basic Principles

- Based on the equivalence suggested by Lambek in 1980

  simply-typed λ-calculus $\Leftrightarrow$ Cartesian Closed Categories

- Translation inspired the *de Bruijn notation* for λ-calculus
- Typing context as left-nested pairs – $\Gamma ::= \star \mid \langle \Gamma, x : A \rangle$
- Each variable is replaced by the path to its position in the context.

$$\text{path}(\langle c, y \rangle, x) = \begin{cases} \text{snd} & \text{if } x = y \\ \text{path}(c, x) \circ \text{fst} & \text{otherwise} \end{cases}$$

Translation Rules

$$
\begin{array}{lcl}
\Phi(\Gamma \vdash \star) & = & \mathsf{bang} \\
\Phi(\Gamma \vdash x) & = & \mathsf{path}(\Gamma, x) \\
\Phi(\Gamma \vdash M\ N) & = & \mathsf{ap} \circ (\Phi(\Gamma \vdash M) \vartriangle \Phi(\Gamma \vdash N)) \\
\Phi(\Gamma \vdash \lambda x.M) & = & \overline{\Phi(\langle \Gamma, x \rangle \vdash M)} \\
\Phi(\Gamma \vdash \langle M, N \rangle) & = & \Phi(\Gamma \vdash M) \vartriangle \Phi(\Gamma \vdash N) \\
\Phi(\Gamma \vdash \mathsf{fst}\ M) & = & \mathsf{fst} \circ \Phi(\Gamma \vdash M) \\
\Phi(\Gamma \vdash \mathsf{snd}\ M) & = & \mathsf{snd} \circ \Phi(\Gamma \vdash M) \\
\Phi(\Gamma \vdash \mathsf{inl}\ M) & = & \mathsf{inl} \circ \Phi(\Gamma \vdash M) \\
\Phi(\Gamma \vdash \mathsf{inr}\ M) & = & \mathsf{inr} \circ \Phi(\Gamma \vdash M) \\
\Phi(\Gamma \vdash \mathsf{case}\ L\ M\ N) & = & \mathsf{ap} \circ (\mathsf{either} \circ (\Phi(\Gamma \vdash M) \\
 & & \qquad \vartriangle \Phi(\Gamma \vdash N)) \vartriangle \Phi(\Gamma \vdash L)) \\
\Phi(\Gamma \vdash \mathsf{in}\ M) & = & \mathsf{in} \circ \Phi(\Gamma \vdash M) \\
\Phi(\Gamma \vdash \mathsf{out}\ M) & = & \mathsf{out} \circ \Phi(\Gamma \vdash M)
\end{array}
$$

Final Details

Unpoint

- Translation of $M : A \to B$
  $N : 1 \to (A \to B)$
- Conversion to expected type $A \to B$
  $\mathsf{ap} \circ (\Phi(\star \vdash M) \circ \mathsf{bang} \vartriangle \mathsf{id})$

Translation of swap

$\mathsf{ap} \circ (\overline{\mathsf{snd} \circ \mathsf{snd} \vartriangle \mathsf{fst} \circ \mathsf{snd}} \circ \mathsf{bang} \vartriangle \mathsf{id})$

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Translating Recursive Definitions
Two Methods

1st Method

- fix f viewed as a stream of functions f (f (f (f ...)))
- Stream $A = \mu(\underline{A} \otimes \mathrm{Id})$
- Hylomorphism that builds tree with in $(f, \mathrm{in}\ (f, ...))$ and replaces in by ap.

$$\begin{array}{lcl} \mathrm{fix} & : & (A \to A) \to A \\ \mathrm{fix} & = & \mathrm{hylo}_{\mathrm{Stream}\ (A \to A)}\ \mathrm{ap}\ (\mathrm{id} \vartriangle \mathrm{id}) \end{array}$$

$$\Phi(\Gamma \vdash \mathrm{fix}\ M) = \mathrm{fix} \circ \Phi(\Gamma \vdash M)$$

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- Algorithm that derives *pointwise hylomorphisms* from explicitly recursive definitions.
- Intermediate data structure models its recursion tree.
- Proposed by Hu, Iwasaki, and Takeichi, in 1996.

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Example

length

```
len :: [a] -> Nat
len []     = Zero
len (h:t) = Succ (len t)
```

$$
\begin{array}{lcl}
\mathsf{length} & : & \mathsf{List}\ A \rightarrow \mathsf{Nat} \\
\mathsf{length} & = & \mathsf{hylo}_{\mu(\underline{1}\oplus \mathsf{Id})} \\
 & & \quad (\lambda x.\ \mathsf{case}\ x\ (\lambda y.\mathsf{in}\ (\mathsf{inl}\ \star))\ (\lambda y.\ \mathsf{in}\ (\mathsf{inr}\ y))) \\
 & & \quad (\lambda x.\ (\mathsf{out}\ x)\ (\lambda y.\ \mathsf{inl}\ \star)\ (\lambda y.\ \mathsf{inr}\ (\mathsf{snd}\ y)))
\end{array}
$$

Outline

4. Pointless Haskell
   - Basic Combinators
   - Recursion
5. DrHylo
   - Pattern Matching
   - *Pointwise* to *Point-free*
   - Recursion
6. SimpliFree
   - Basic Principles
   - Rule Construction
   - Strategies
7. Conclusions

Main Topics

- *Active source* – Manipulated code with special commented blocks;
- Rules and strategies can be defined;
- Implemented by generic traversals – with *Strafunski*;
- Uses *Haskell* pattern matching;
- Uses parameterized macros.

Tool Interacion

Input

- *Point-free* functions;
- Special annotated blocks
  - Import strategy pack;
  - Rules;
  - Strategies;
  - Optimisations (functions $\rightharpoonup$ strategies).

Output

- Strategies $\rightarrow$ special combinators;
- Rules $\rightarrow$ functions with type `Term -> Maybe Term`;
- Each optimized function $\rightarrow$ Computation;
- `main` function $\rightarrow$ Transformed file.

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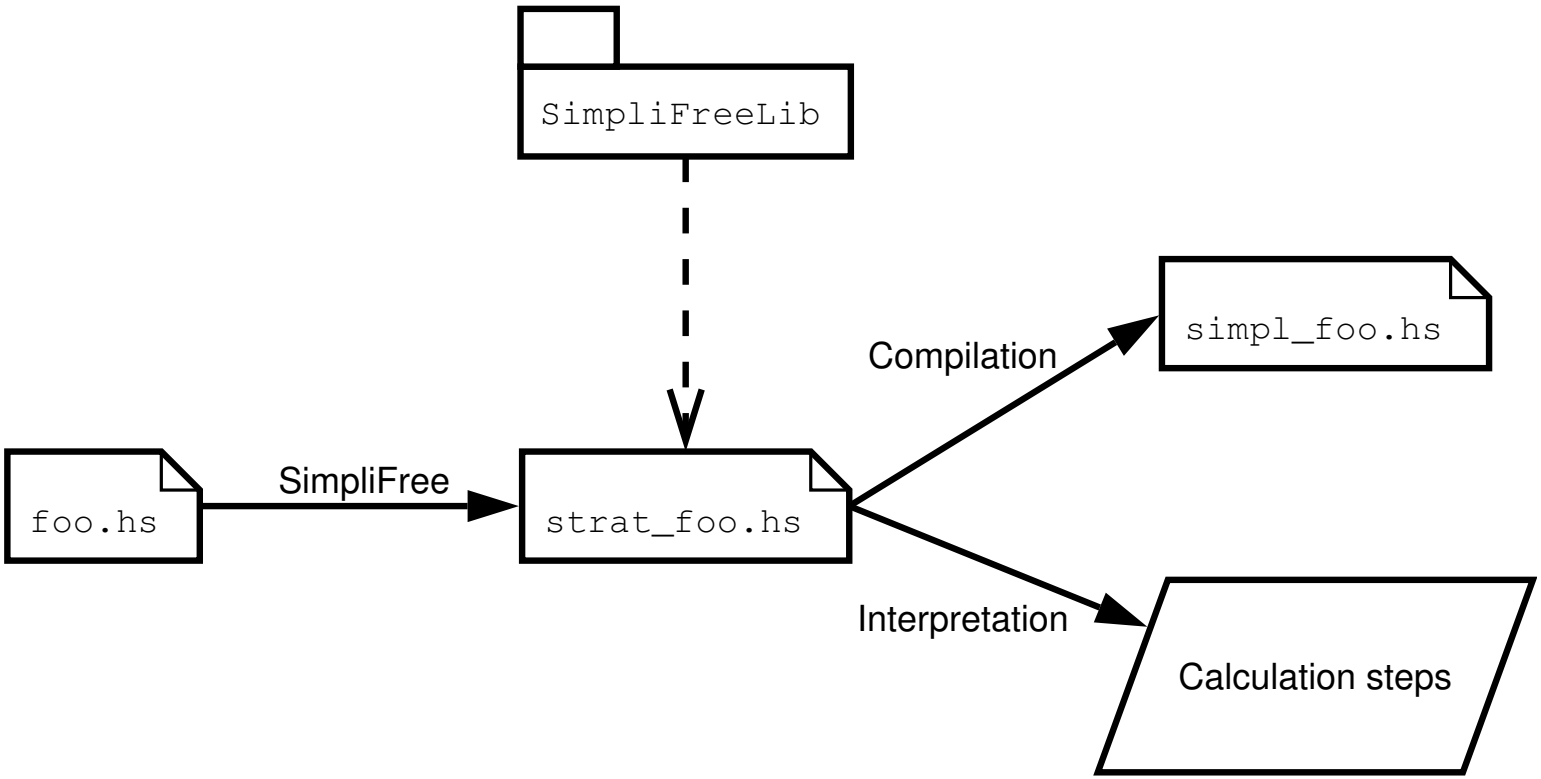
SimpliFree Architecture



Rule Construction

```
prodCancel : fst . (f /\ g) -> f
```



```
prodCancel (FST :.: (f :/\: g)) = return f
prodCancel (FST :.: ((f :/\: g) :.: x))
   = return (f :.: x)
prodCancel _
   = fail "rule prodCancel1 not applied"
```

Solved problems

- Variables on the left of a composition and correct association;
- Condition verification may require backtracking;
- Rules produced automatically

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Defined Strategies

Rules repository with some rules and strategies.
Most depend on a base strategy.

Base Strategy

1. Apply simplification rules while possible;
2. If there are known macros, unfold them and return to (1);
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Addition of new simplification strategies and macros.

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Examples
Swap

```
*Main> swap_adv_strat
app.((curry ((snd.snd) /\ (fst.snd)).bang)
                                        /\ id)
  = { expCancAdv3 }
((snd.snd) /\ (fst.snd)).(bang /\ id)
  = { prodFus }
(snd.snd.(bang /\ id)) /\ (fst.snd.(bang /\ id))
  = { prodCancel2 }
(snd.id) /\ (fst.snd.(bang /\ id))
  = { natId2 }
snd /\ (fst.snd.(bang /\ id))
  = { prodCancel2 }
snd /\ (fst.id)
  = { natId2 }
snd /\ fst
  = { swap_fold }
'swap'
```

Examples
cata-Fusion

```
*Main> reverse_t_cataList_strat
curry 'cat'.('cataList' [('pnt' ['nil']) \/
                ('cat'.'swap'.('wrap' >< id))])
= { cataList
    --- and ---
    [curry 'cat',curry 'cat'.'cat'.'swap'.('wrap' >< id)]
      = { catAssoc }
    [curry 'cat','comp'.(curry 'cat' >< curry 'cat').'swap'.
                                                ('wrap' >< id)]
      = { swapLeft }
    [curry 'cat','comp'.'swap'.(curry 'cat' >< curry 'cat').
                                                ('wrap' >< id)]
      = { prodFun }
    [curry 'cat','comp'.'swap'.((curry 'cat'.'wrap') ><
                                          (curry 'cat'.id))]
      = { natId2 }
    [curry 'cat','comp'.'swap'.((curry 'cat'.'wrap') ><
                                            curry 'cat')]
      = { extractH2B }
    'comp'.'swap'.((curry 'cat'.'wrap') >< id)
  }
'cataList' [(curry 'cat'.('pnt' ['nil'])) \/
       ('comp'.'swap'.((curry 'cat'.'wrap') >< id))]
```

Outline

Conclusions and Future Work

- **Pointless** – reached a stable stage;
- ***DrHylo*** – translate to other recursion patterns
- **SimpliFree**
  – incorporate other laws for recursive patterns
  – make the fusion mechanism more powerful
  – make laws more generic, like the fold fusion

- More components of the framework can be found in the UMinho Haskell Software pages.

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