



Exercises 2 : Processos e Concorrência

Luís Soares Barbosa

Exercise I.1

Suppose a labelled transition system is given by the following transition relation:

$$\{\langle 1, a, 2 \rangle, \langle 1, a, 3 \rangle, \langle 2, a, 3 \rangle, \langle 2, b, 1 \rangle, \langle 3, a, 3 \rangle, \langle 3, b, 1 \rangle, \langle 4, a, 5 \rangle, \langle 5, a, 5 \rangle, \langle 5, b, 6 \rangle, \langle 6, a, 5 \rangle, \langle 7, a, 8 \rangle, \langle 8, a, 8 \rangle, \langle 8, b, 7 \rangle\}$$

Prove or refute $1 \sim 4 \sim 6 \sim 7$.

Exercise I.2

Given two labelled transition systems $\langle S_A, \mathcal{N}, \downarrow_A, \rightarrow_A \rangle$ and $\langle S_B, \mathcal{N}, \downarrow_B, \rightarrow_B \rangle$, two states p and q are *mutually similar* iff

$$p \doteq q \equiv p \lesssim q \wedge q \lesssim p$$

1. Show that \doteq is an equivalence relation.
2. Compare this equivalence with bisimilarity \sim .

Exercise I.3

Show that \sim is an equivalence relation.

Exercise I.4

Discuss whether bisimilarity \sim

- is closed for union
- is closed for intersection

Exercise I.5

A relation R over the state space of a labelled transition system is a *word bisimulation* if, whenever $\langle p, q \rangle \in R$ and $s \in \mathcal{N}^*$, we have

$$\begin{aligned} p \xrightarrow{s} p' &\Rightarrow \langle \exists q' : q' \in S_2 : q \xrightarrow{s} q' \wedge \langle p', q' \rangle \in R \rangle \\ q \xrightarrow{s} q' &\Rightarrow \langle \exists p' : p' \in S_1 : p \xrightarrow{s} p' \wedge \langle p', q' \rangle \in R \rangle \end{aligned}$$

1. Define formally relation \xrightarrow{s} , for $s \in \mathcal{N}^*$
2. Two states are *word bisimilar* iff they belong to a word bisimulation. Show that two states p and q are word bisimilar iff $p \sim q$.