



## Exercises 5: Processos e Concorrência

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### Exercício I.1

Suppose two variants of parallel composition have been added to the process language  $\mathbb{P}$  and defined through the following rules:

$$\frac{E \xrightarrow{a} E'}{E \otimes F \xrightarrow{a} E' \otimes F} (O_1)$$

$$\frac{F \xrightarrow{a} F'}{E \otimes F \xrightarrow{a} E \otimes F'} (O_2)$$

$$\frac{E \xrightarrow{a} E' \quad \wedge \quad \bar{a} \notin \mathcal{L}(F)}{E \parallel F \xrightarrow{a} E' \parallel F} (P_1)$$

$$\frac{F \xrightarrow{a} F' \quad \wedge \quad \bar{a} \notin \mathcal{L}(E)}{E \parallel F \xrightarrow{a} E \parallel F'} (P_2)$$

$$\frac{E \xrightarrow{a} E' \quad F \xrightarrow{\bar{a}} F'}{E \parallel F \xrightarrow{\tau} E' \parallel F'} (P_3)$$

1. Explain, in your own words, the meaning of  $\otimes$  e  $\parallel$ .
2. Guided by the semantic rules given, show how the synchronisation diagrams for  $E \otimes F$  and  $E \parallel F$  can be built from the corresponding diagrams for  $E$  and  $F$ .
3. Is  $\parallel$  associative with respect to  $\sim$ ?

### Exercício I.2

Identify, in the list of process pairs below, which of them can be related by  $\approx$ . And by  $=$ ?

1.  $a.\tau.b.\mathbf{0} \text{ e } a.b.\mathbf{0}$
2.  $a.(b.\mathbf{0} + \tau.c.\mathbf{0}) \text{ e } a.(b.\mathbf{0} + c.\mathbf{0})$
3.  $a.(b.\mathbf{0} + \tau.c.\mathbf{0}) \text{ e } a.(b.\mathbf{0} + c.\mathbf{0}) + a.c.\mathbf{0}$
4.  $a.\mathbf{0} + b.\mathbf{0} + \tau.b.\mathbf{0} \text{ e } a.\mathbf{0} + \tau.b.\mathbf{0}$
5.  $a.\mathbf{0} + b.\mathbf{0} + \tau.b.\mathbf{0} \text{ e } a.\mathbf{0} + b.\mathbf{0}$
6.  $a.(b.\mathbf{0} + (\tau.(c.\mathbf{0} + \tau.d.\mathbf{0}))) \text{ e } a.(b.\mathbf{0} + (\tau.(c.\mathbf{0} + \tau.d.\mathbf{0}))) + a.(c.\mathbf{0} + \tau.d.\mathbf{0})$
7.  $a.(b.\mathbf{0} + (\tau.(c.\mathbf{0} + \tau.d.\mathbf{0}))) \text{ e } a.(b.\mathbf{0} + c.\mathbf{0} + d.\mathbf{0}) + a.(c.\mathbf{0} + d.\mathbf{0}) + a.d.\mathbf{0}$
8.  $\tau.(a.b.\mathbf{0} + a.c.\mathbf{0}) \text{ e } \tau.a.b.\mathbf{0} + \tau.a.c.\mathbf{0}$
9.  $\tau.(a.\tau.b.\mathbf{0} + a.b.\tau.\mathbf{0}) \text{ e } a.b.\mathbf{0}$
10.  $\tau.(a.\tau.\mathbf{0} + \tau.b.\mathbf{0}) \text{ e } \tau.a.\mathbf{0} + \tau.b.\mathbf{0}$
11.  $A \triangleq a.\tau.A \text{ e } B \triangleq a.B$
12.  $A \triangleq \tau.A + a.\mathbf{0} \text{ e } a.\mathbf{0}$
13.  $A \triangleq \tau.A \text{ e } \mathbf{0}$

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**Exercício I.3**

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Suppose processes  $R$  and  $T$  have transitions  $R \xrightarrow{\tau} T$  and  $T \xrightarrow{\tau} R$ , among others. Show that, under this condition,  $R = T$ .

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**Exercício I.4**

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Consider the following statements about a binary relation  $S$  on  $\mathbb{P}$ . Discuss whether you may conclude from each of them whether  $S$  is (or is not) a weak bisimulation.

observacional:

1.  $S$  is the identity in  $\mathbb{P}$ .
  2.  $S$  is a subset of the identity in  $\mathbb{P}$ .
  3.  $S$  is a strict bisimulation up to  $\equiv$ .
  4.  $S$  is the empty relation.
  5.  $S = \{(a.E, a.F) \mid E \approx F\}$ .
  6.  $S = \{(a.E, a.F) \mid E \approx F\} \cup \approx$ .
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**Exercício I.5**

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Show that

1.  $E + \tau.(E + F) = \tau.(E + F)$
  2.  $a.(E + \tau.\tau.E) = a.E$
  3.  $\tau.(G + a.(E + \tau.F)) = \tau.(G + a.(E + \tau.F)) + a.F$
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**Exercício I.6**

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Show that any process  $\tau.(\tau.P + a.0)$  is a solution to equation  $X = a.0 + \tau.X$ .

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**Exercício I.7**

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Let  $E$  be a process such that  $\text{fn}(E) = \emptyset$ . Prove or refute the following statements:

1.  $E \mid Q \approx Q$ .
  2.  $E \mid Q = Q$ .
  3.  $E \mid Q = \tau.Q$ .
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**Exercício I.8**

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Although concurrent systems usually deal with components exhibiting non terminating behaviour, it is sometimes useful also to consider terminating processes and their composition. Let  $T$  be a class of terminating processes which perform a special action  $\dagger$  to announce completion of all their tasks and evolve to  $0$  after that. In this class it is possible to define a combinator for *sequential* composition  $P ; Q$ , whose behaviour is informally explained as *once  $P$  terminates,  $P ; Q$  behaves like  $Q$* . Formally,

$$P ; Q \triangleq \text{new } \{m\} (\{m/\dagger\} P \mid \bar{m} \cdot Q)$$

where  $m$  is fresh identifier, not occurring neither in  $P$  nor  $Q$ .

1. Define a process  $U \in T$  such that  $U ; P \approx P$ . Justify your proposal.
2. Prove or refute that, for any  $P, Q, R \in T$ ,
 
$$(P + Q) ; R \approx (P ; R) + (Q ; R)$$
3. As sequential composition is a particular case of parallel composition, the law above could be regarded as a particular case of
 
$$(P + Q) | R \approx (P | R) + (Q | R)$$
 This equation, however, is false. Confirm this by providing a suitable counter-example..

### Exercício I.9

Consider the following specification of a *pipe*, as supported e.g. in UNIX:

$$U \triangleright V \stackrel{\text{abv}}{=} \text{new } \{c\} (\{c/\text{out}\}U | \{c/\text{in}\}V)$$

under the assumption that, in both processes, actions  $\overline{\text{out}}$  e  $\text{in}$  stand for, respectively, the output and input ports.

1. Consider now the following processes only partially defined:

$$\begin{aligned} U_1 &\triangleq \overline{\text{out}}.T \\ V_1 &\triangleq \text{in}.R \\ U_2 &\triangleq \overline{\text{out}}.\overline{\text{out}}.\overline{\text{out}}.T \\ V_2 &\triangleq \text{in}.\text{in}.\text{in}.R \end{aligned}$$

Prove, by equational reasoning, or refute the following properties:

- (a)  $U_1 \triangleright V_1 \sim T \triangleright R$
  - (b)  $U_2 \triangleright V_2 = U_1 \triangleright V_1$
2. Show or refute the associativity of  $\triangleright$  wrt process equality, i.e., for all  $P, T, V \in \mathbb{P}$ ,
 
$$(U \triangleright V) \triangleright T = U \triangleright (V \triangleright T)$$
  3. Show that  $\mathbf{0} \triangleright \mathbf{0} = \mathbf{0}$ .

### Exercício I.10

Consider a combinator  $\circ_n$  whose operational semantics is given by following rule

$$\frac{E \xrightarrow{a} E'}{\circ_0 E \xrightarrow{a} E'} \quad \frac{E \xrightarrow{a} E'}{\circ_n E \xrightarrow{a} \circ_{n-1} E'} \quad \text{for } n > 0$$

1. Explain its purpose.
2. Discuss whether, and for which values of  $m$  and  $n$ , one may have  $\circ_n (\circ_m E) \sim \circ_n E$ .
3. Show that  $E \sim F$  implies  $\circ_n E \sim \circ_n F$ .
4. Show, by a counter-example, that, whenever  $\sim$  is replaced by  $\approx$ , the implication above fails.
5. How could the operational semantics of this new combinator be changed so that the implication mentioned above holds? I.e. so that  $E \approx F \Rightarrow \circ_n E \approx \circ_n F$ ?

### Exercício I.11

Consider a combinator whose operational semantics is given by following rule

$$\frac{E \xrightarrow{x} E'}{E \downarrow a \xrightarrow{x} E'} \quad \text{if } x \neq a, x \neq \bar{a}$$

1. Explain its purpose.
2. Show that  $P \downarrow a \sim Q \downarrow a$  if  $P \sim Q$ .
3. Define two processes  $E$  and  $F$  such that  $E \approx F$  but  $E \downarrow a \not\approx F \downarrow a$ .
4. Prove or refute that if  $P = Q$  then  $P \downarrow a = Q \downarrow a$ .

### Exercício I.12

Consider a new process combinator, called an *action duplicator*, and defined by the following rule:

$$\frac{E \xrightarrow{a} E'}{\circ(E) \xrightarrow{a} E}$$

Note that the derivative in the rule's conclusion is  $E$  (and not  $E'$ ). For example,  $\circ(a.\mathbf{0}) \xrightarrow{a} a.\mathbf{0}$ . Prove or refute that

1.  $E \sim F$  implies  $\circ(E) \sim \circ(F)$ .
2.  $E \approx F$  implies  $\circ(E) \approx \circ(F)$ .
3.  $\circ(E + F) \sim \circ(E) + \circ(F)$ .