

Bisimulation

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Simulation

the quest for a **behavioural equality**:
able to identify states that cannot be distinguished by any **realistic**
form of observation

Simulation

A state q **simulates** another state p if every transition from q is corresponded by a transition from p and this capacity is kept along the whole life of the system to which state space q belongs to.

Simulation

Definition

Given $\langle S_1, N, \downarrow_1, \longrightarrow_1 \rangle$ and $\langle S_2, N, \downarrow_2, \longrightarrow_2 \rangle$ over N , relation $R \subseteq S_1 \times S_2$ is a **simulation** iff, for all $\langle p, q \rangle \in R$ and $a \in N$,

$$(1) \quad p \downarrow_1 \Rightarrow q \downarrow_2$$

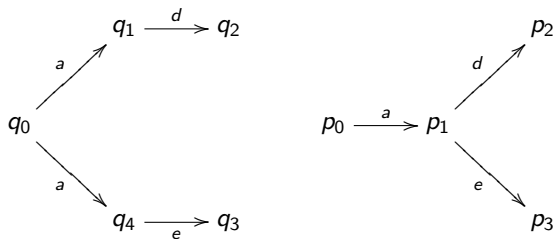
$$(2) \quad p \xrightarrow{a}_1 p' \Rightarrow \langle \exists q' : q' \in S_2 : q \xrightarrow{a}_2 q' \wedge \langle p', q' \rangle \in R \rangle$$

$$\begin{array}{c} p \quad R \quad q \\ \downarrow a \\ p' \end{array}$$

 \Rightarrow

$$\begin{array}{c} q \\ \downarrow a \\ p' \quad R \quad q' \end{array}$$

Example



$$q_0 \lesssim p_0 \quad \text{cf.} \quad \{\langle q_0, p_0 \rangle, \langle q_1, p_1 \rangle, \langle q_4, p_1 \rangle, \langle q_2, p_2 \rangle, \langle q_3, p_3 \rangle\}$$

Similarity

Definition

$$p \lesssim q \Leftrightarrow \langle \exists R :: R \text{ is a simulation and } \langle p, q \rangle \in R \rangle$$

Lemma

The similarity relation is a preorder
(ie, reflexive and transitive)

Bisimulation

Definition

Given $\langle S_1, N, \downarrow_1, \longrightarrow_1 \rangle$ and $\langle S_2, N, \downarrow_2, \longrightarrow_2 \rangle$ over N , relation $R \subseteq S_1 \times S_2$ is a **bisimulation** iff both R and its converse R° are simulations.

I.e., whenever $\langle p, q \rangle \in R$ and $a \in N$,

$$(1) \quad p \downarrow_1 \Leftrightarrow q \downarrow_2$$

$$(2.1) \quad p \xrightarrow{a}_1 p' \Rightarrow \langle \exists q' : q' \in S_2 : q \xrightarrow{a}_2 q' \wedge \langle p', q' \rangle \in R \rangle$$

$$(2.1) \quad q \xrightarrow{a}_2 q' \Rightarrow \langle \exists p' : p' \in S_1 : p \xrightarrow{a}_1 p' \wedge \langle p', q' \rangle \in R \rangle$$

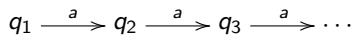
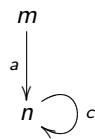
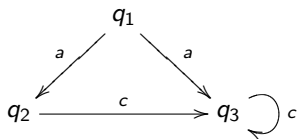
Bisimulation

The Game characterization

Two players R and I discuss whether the transition structures are mutually corresponding

- R starts by choosing a transition
- I replies trying to match it
- if I succeeds, R plays again
- R wins if I fails to find a corresponding match
- I wins if it replies to all moves from R and the game is in a configuration where all states have been visited or R can't move further. In this case is said that I has a **wining strategy**

Examples



Bisimilarity

Definition

$$p \sim q \Leftrightarrow \langle \exists R :: R \text{ is a bisimulation and } \langle p, q \rangle \in R \rangle$$

Lemma

1. The identity relation id is a bisimulation
2. The empty relation \perp is a bisimulation
3. The converse R° of a bisimulation is a bisimulation
4. The composition $S \cdot R$ of two bisimulations S and R is a bisimulation
5. The $\bigcup_{i \in I} R_i$ of a family of bisimulations $\{R_i \mid i \in I\}$ is a bisimulation

Bisimilarity

Lemma

The bisimilarity relation is an equivalence relation
(ie, reflexive, symmetric and transitive)

Lemma

The class of all bisimulations between two LTS has the structure of a **complete lattice**, ordered by set inclusion, whose top is the **bisimilarity** relation \sim .

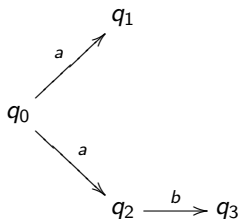
Bisimilarity

Warning

The bisimilarity relation \sim is not the symmetric closure of \lesssim

Example

$$q_0 \lesssim p_0, p_0 \lesssim q_0 \quad \text{but} \quad p_0 \not\sim q_0$$



$$p_0 \xrightarrow{a} p_1 \xrightarrow{b} p_3$$

Notes

Similarity as the greatest simulation

$$\lesssim \triangleq \bigcup \{S \mid S \text{ is a simulation}\}$$

Bisimilarity as the greatest bisimulation

$$\sim \triangleq \bigcup \{S \mid S \text{ is a bisimulation}\}$$

cf **relational** translation of definitions
 \lesssim and \sim as **greatest fix points** (Tarski's theorem)