



Exercises 4 : Interaction and Concurrency

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Exercise I.1

Let $A(a) \triangleq a.A$ and $B(b) \triangleq \bar{b}.B$. Compute the first derivatives of the following processes:

1. $A + B$
 2. $A + B\langle a \rangle$
 3. $A \mid B$
 4. $A \mid B\langle a \rangle$
 5. $\{a/b\}(A \mid B)$
 6. $\text{new } \{a\}(A \mid B\langle a \rangle)$
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Exercise I.2

Let $A(a, b, c, d) \triangleq \bar{a}.b.A + \bar{c}.d.A$. Draw the transition graphs of the following processes

1. A
 2. $\text{new } \{a\} A$
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Exercise I.3

Consider the following description of a two-position *buffer* with acknowledgements. Note the process is built from copies of a 1-position *buffer* also with acknowledgements: it acknowledges in \bar{r} the reception of a message and waits in t the confirmation that a message sent was arrived to its destination.

$$Bs \triangleq \text{new } \{mo, mi\} (B(in, mo, mi, r) \mid B(mo, out, t, mi))$$
$$B(in, out, t, r) \triangleq in.\overline{out}.t.\bar{r}.B$$

1. Draw the synchronisation graph of Bs .
2. Check whether the behaviour of Bs is the intended one (drawing, for this purpose, the corresponding transition graph)
3. Find a solution to the problem detected (if any) and draw the corresponding transition graph.
4. Explain how the specification given (or your new solution) can be adapted to describe *buffers* with an arbitrary, but fixed number of positions.

Exercise I.4

Consider the following description of a 1-position bidirectional buffer, *i.e.*, able to transmit and receive messages in any direction.

$$BT(in_1, in_2, out_1, out_2) \triangleq in_1(x).\overline{out_1}(x).BT + in_2(x).\overline{out_2}(x).BT$$

1. Specify a 2-position bidirectional buffer by parallel composition of two instances of process BT .
 2. Draw its synchronisation diagram and the transition graph.
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Exercise I.5

Consider the following specification of a control system for a crossing between a road and a railway. Events car and $train$ modelled, respectively, a car or a train approaching the cross. Actions up e dw stand for the opening and closing of the protection bar to prevent cars to cross. Similarly, $green$ and red model the semaphore for trains. Finally, events \overline{ccross} and \overline{tcross} come from sensors which register the actual cross of a car or a train, respectively.

$$\begin{aligned} Road &\triangleq car.up.\overline{ccross}.dw.Road \\ Rail &\triangleq train.green.\overline{tcross}.red.Rail \\ Signal &\triangleq \overline{green}.red.Signal + \overline{up}.dw.Signal \end{aligned}$$

$$C \triangleq \text{new } \{green, red, up, dw\} (Road \mid Rail \mid Signal)$$

1. Explain the behaviour of this process and sketch its synchronisation diagram.
 2. Compute the transition graph corresponding to process C
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Exercise I.6

An n -trigger, for $n > 1$, is used in electronic voting to detect that a fixed number of votes have been received along its n input ports, numbered from a_1 to a_n . As soon votes have been received in half of the input ports a signal is sent through its output port \bar{s} and the process terminates. Each port a_i receives only a single input. Inputs, however, may arrive in any order to the different ports.

1. Specify a 3-trigger.
 2. Specify a n -trigger, for n arbitrary.
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Exercise I.7

Draw the transition graph of $T \triangleq a.(b.0 \mid T)$?