Program Semantics, Verification, and Construction - Part I
MAP-i 2007-2008
Due date: 15/02/2008

1. (a) Write an extension of the simple type system for the λ-calculus, for a term language consisting of the λ-calculus, integers, arithmetic operators $(+, -, \times)$, *if then else*, pairs, projections (*first* e *second*), local variable definitions *let* $x = t_1$ *in* t_2, where the scope of x is t_2 (note that this expression is semantically equivalent to $(\lambda x.t_2)t_1$) and a *rec* operator $y.(\lambda x.t)$, where the function $y = \lambda x.t$ is defined recursively (i.e. y occurs free in t).

   (b) Define an operational semantics to the previous language using call by value. Values are integers, pairs of values and close terms of the form $\lambda x.t$.

   (c) Show that, for the semantics and type system of the previous questions, if $t \to c$ and t is typed by τ then c is also typed by τ.

2. Write a small essay ($\approx$ 1000 words) about **one** of the following topics:

   (a) Curry-Howard correspondence for combinatory logic and Hilbert-style axioms of intuitionistic propositional logic (implicational fragment).
   Bibliography: [GLM97] [Hin97] [PU96]

   (b) Curry-Howard correspondence for intuitionistic propositional logic presented in sequent calculus. Discuss if we have a one-to-one map.
   Bibliography: [PU96] [BG02]

   (c) Curry-Howard correspondence for classical propositional logic.
   Bibliography: [PU96]

   (d) Curry-Howard correspondence for substructural propositional logics $R_{\rightarrow}$, **BCK** and **BCI** (implicational fragments).
   Bibliography: [Hin97] [PU96]

References

[BG02] Henk Barendregt and Sylvia Ghilezan. Lambda terms for natural dedution, sequent calculus and cut elimination. *J. of Functional Programming*, 2002.

[GLM97] Jean Goubault-Larrecq and Ian Mackie. *Proof Theory and Automated Deduction*. Kluwer Academic Press, 1997.

[Hin97] J. Roger Hindley. *Basic simple type theory*. Number 42 in Cambridge Tracts in Theoretical Computer Science. CUP, 1997.

[PU96] Morten B. Sorensen Pawel Urzyczyn. Lecture on the curry-howard isomorphism. Technical report, University of Copenhagen, 1996. http://zls.mimuw.edu.pl/ urzy/ftp.html.