



# A Taste of Program Verification

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#### Outline

- Formal models and the central problem of formal methods
- Introduction to Hoare Logic; verification by hand
- Specifying the behaviour of C programs
- Case study: array partition algorithm



#### Outline

- Program Annotation and Design by Contract
- JML Tool Demo: ESC/Java2 + Simplify
- Tool Demo: Caduceus + Coq



# The Central Problem of Formal Methods



## Models: Tools and Approaches

- Abstract State Machines (B)
- Automata-based Models
- Process Algebra (Esterel)
- Set and Category Theory (Z,VDM, Charity)
- Algebraic Specifications (OBJ)
- Declarative Modeling (FP, LP, TRS)
- Preconditions and PostConditions





#### The Central Problem of FM

#### Part I: model validation

- How to enforce, at the specification level, the desired behaviour?

Prove properties about the model





# Tools for Formal Verification

Proof Systems:

Theorem Provers / Proof Assistants

- Model Checkers



#### The Central Problem of FM

#### Part 2: relation between specifications and implementations

- Given an implementation, how can it be checked that it obeys the specification?
   Testing; Program Verification





#### Program Extraction

 From a proof of a logical property (typically concerning existential quantifications), the Coq system is capable of extracting a program into a working programming language



#### Program Derivation

- Stepwise Refinement from Specifications to Programs (Z,VDM, B, ...)
- Two approaches to correctness:
  - (i) the refinement steps generate *proof obligations* that must be discharged. Derivations are thus formally verified.
  - (ii) the refinement process is itself verified to be correct. The derived programs are then *correct by construction*.





## Program Verification

- Given a program and a specification, check that the former conforms to the latter.
- This is the only applicable method in many situations
- THIS LECTURE: an approach to program verification based on program annotation and Hoare Logic





# Hoare Logic





- A formal system that is useful for
  - Correct by construction program derivation extensive bibliography: Kaldewaij; Gries; Backhouse; Dijkstra
  - Our focus: Program Verification
- Formulas assert that if a given precondition holds prior to program execution, then a postcondition will hold after execution



# A Toy Programming Language

#### Types (data and expressions):

```
egin{array}{lll} 	au &::= & \mathbf{bool} \mid \mathbf{int} \ 	heta &::= & \mathbf{var} \mid \mathbf{exp}[	au] \mid \mathbf{com} \mid \mathbf{assert} \end{array}
```

#### Interpreted as expected:

$$[\![\mathbf{bool}]\!] = \{true, false\}$$
$$[\![\mathbf{int}]\!] = \{\dots -2, -1, 0, 1, 2, \dots\}$$



# Espressions, commands and assertions are interpreted in a given state of the program

$$\mathcal{I} = \{x, y, z, \ldots\}$$

$$\Sigma = \mathcal{I} o \llbracket \mathbf{int} 
rbracket$$

$$\llbracket \exp[\tau] \rrbracket = \Sigma \to \llbracket \tau \rrbracket_{\perp}$$

$$[\![\mathbf{com}]\!] = \Sigma \to \Sigma_{\perp}$$

$$[assert] = \Sigma \rightarrow \{true, false\}$$

# **Abstract Syntax**

```
V ::= x, y, z, \dots
                                 var, int, exp[bool], exp[int], com, and assert
B ::= true \mid false
      | B \&\&B | B ||B| !B
      | E == E | E < E | E <= E | E > E | E >= E | E! = E
E ::= \ldots -2, -1, 0, 1, 2 \ldots \mid V \mid L
      -E \mid E + E \mid E - E \mid E * E \mid E \operatorname{\mathbf{div}} E \mid E \operatorname{\mathbf{mod}} E
C ::= \mathbf{skip} \mid C; C \mid V := E \mid \mathbf{if} \ E \ \mathbf{then} \ C \ \mathbf{else} \ C \mid \mathbf{while} \ (E) \ \mathbf{do} \ C
A ::= true \mid false
      | A \&\&A | A | |A| | |A| | \forall L.A | \exists L.A | A \rightarrow A
           E == E \mid E < E \mid E <= E \mid E > E \mid E >= E \mid E! = E
```

#### Interpretation of Expressions

#### Interpretation of Commands

$$[\![skip]\!](s) = s$$

$$[\![C_1; C_2]\!](s) = ([\![C_2]\!] \odot [\![C_1]\!])(s)$$

$$\text{where } [\![g \odot f]\!](s) = \begin{cases} \bot & \text{if } [\![f]\!](s) = \bot \\ g(f(s)) & \text{otherwise} \end{cases}$$

$$[\![x := E]\!](s) = \begin{cases} s[x \mapsto [\![E]\!](s)] & \text{if } [\![E]\!](s) \neq \bot \\ \text{otherwise} \end{cases}$$

$$[\![\mathbf{if}\ E_1\ \mathbf{then}\ E_2\ \mathbf{else}\ E_3]\!](s) = \operatorname{cond}([\![E_1]\!], [\![E_2]\!], [\![E_3]\!])(s)$$

$$\operatorname{and}\operatorname{cond}(p, f, g)(s) = \begin{cases} f(s) & \text{if } p(s) = true \\ g(s) & \text{if } p(s) = false \end{cases}$$

$$\perp \quad \text{otherwise}$$

# Interpretation of Assertions (I)

```
[true](s) = true
        [false](s) = false
            \llbracket !a \rrbracket(s) = \begin{cases} true & \text{if } \llbracket e \rrbracket(s) = false \\ false & \text{if } \llbracket e \rrbracket(s) = true \end{cases}
 [a_1 \&\& a_2](s) = [a_1](s) \text{ and } [a_2](s)
     [a_1 | a_2](s) = [a_1](s) \text{ or } [a_2](s)
 [a_1 \to a_2](s) = \text{if } [a_1](s) \text{ then } [a_2](s)
   [e_1 < e_2](s) = \begin{cases} [e_1](s) < [e_2](s) & \text{if } [e_1](s) \neq \bot \text{ and } [e_2](s) \neq \bot \\ false & \text{otherwise} \end{cases}
[e_1 == e_2](s) = \begin{cases} [e_1](s) == [e_2](s) & \text{if } [e_1](s) \neq \bot \text{ and } [e_2](s) \neq \bot \\ false & \text{otherwise} \end{cases}
```

## Interpretation of Assertions (2)

$$\llbracket \forall x. a \rrbracket(s) = \text{for every } v \in \llbracket \mathbf{int} \rrbracket_{\perp}, \llbracket a \rrbracket(s) [x \mapsto v] = true$$

$$\llbracket\exists x. a \rrbracket(s) = \text{for some } v \in \llbracket \mathbf{int} \rrbracket_{\perp}, \llbracket a \rrbracket(s) [x \mapsto v] = true$$





#### Hoare Triple Formulas

 $\{P\} C \{Q\}$ 

P,Q: assert are closed with respect to logical variables

 $C:\mathbf{com}$  contains no occurrences of logical variables

meaning that if C executes in a state where P holds, then if C terminates Q will hold upon termination



## Semantics of Hoare Triples

Given by the following interpretation in  $\{true, false\}$ , using the semantics of assertions

P, Q may contain occurrences of program variables that do not occur in C. Such variables are called auxiliary



This is a *partial* notion of correctness since the program is not guaranteed to terminate.

If additionally the existence of s is required, we are in the presence of total correctness formulas.

$$\{P\} C \{Q\} \text{ and } C \text{ terminates} \equiv [P] C [Q]$$



# Inference System

An inference system can be defined that derives only valid Hoare triples: if

$$\{P\} C \{Q\}$$

is derived then

$$[\![\{P\}C\{Q\}]\!] = true$$





# Skip and Composition

 $\overline{\{P\}\operatorname{\mathbf{skip}}\{P\}}$ 

$$\frac{\{P\}\,C_1\,\{Q\}}{\{P\}\,C_1;C_2\,\{R\}}$$



# Assignment

#### Works backwards

$$\overline{\{Q[x \mapsto e]\} \, x := e \, \{Q\}}$$

Example:

$${x+1=4} x := x+1 {x=4}$$





#### Conditional

$$\frac{\{P \&\& B\} C_t \{Q\}}{\{P\} \text{ if } B \text{ then } C_t \text{ else } C_f \{Q\}}$$

Can you spot a minor imprecision here?



#### Loops

A fundamental notion: a *loop invariant* is a property that is preserved by the body of a loop, i.e. if it holds as a precondition together with the loop condition then it holds as a post-condition

$$\frac{\{I \&\& B\} C \{I\}}{\{I\} \text{ while } (B) \text{ do } C \{I \&\& \neg B\}}$$

The identification of loop invariants is a crucial task



#### Logical Rules

We also need rules that relate assertions with specifications. Preconditions can be strengthened or made disjuncts

$$\frac{P' \to P \qquad \{P\} C \{Q\}}{\{P'\} C \{Q\}}$$

$$\frac{\{P_1\} C \{Q\} \dots \{P_n\} C \{Q\}}{\{P_1 \mid | \dots | | P_n\} C \{Q\}}$$



#### More Logical Rules

Postconditions can be weakened or made conjuncts

$$\frac{\{P\} C \{Q\} \qquad Q \to Q'}{\{P\} C \{Q'\}}$$

$$\frac{\{P\} C \{Q_1\} \dots \{P\} C \{Q_n\}}{\{P\} C \{Q_1 \&\& \dots \&\& Q_n\}}$$



# More Logical Rules

0-ary cases for conjunction and disjunction

 $\{\mathbf{false}\} C \{Q\}$ 

 $\{Q\} C \{ \mathbf{true} \}$ 



#### Example: Verification by Hand

Take the exponentiation function

$$\exp(x,0) = 1$$

$$\exp(x,n+1) = x * \exp(x,n)$$

We intend to write a program calcexp such that

$$\{$$
**true** $\}$  **calcexp**  $\{$ *w* $=$  exp $(x,y)\}$ 

In fact this needs to be refined with the help of auxiliary variables, not used by calcexp

$$\{x = X_0 \land y = Y_0\} \text{ calcexp } \{w = \exp(x, y) \land x = X_0 \land y = Y_0\}$$

$$\{x = X_0 \land y = Y_0\} \text{ calcexp } \{w = \exp(X_0, Y_0)\}$$



#### The loop condition:

$$B \equiv z \leq y$$

while 
$$z \le y$$
 do

$$W := W * X;$$

$$z := z + 1;$$

#### The loop invariant *P*:

$$P \equiv I \wedge R \wedge w = \exp(X_0, z - 1)$$

$$I \equiv x = X_0 \land y = Y_0$$

$$R \equiv 1 \leq z \leq y+1$$

P will grant the postcondition upon termination



$$P \equiv I \wedge R \wedge w = \exp(X_0, z - 1)$$

Invariant preservation

$$\{P \wedge B\} \ C \ \{P\}$$

Start with assignment axioms

$$\overline{\{I \land 1 \le z + 1 \le y + 1 \land w = \exp(X_0, (z+1) - 1)\}} \ \mathbf{z} := \mathbf{z+1} \ \{P\}$$

$$\overline{\{I \land 0 \le z \le y \land w = \exp(X_0, z)\}} \ \mathbf{z} := \mathbf{z+1} \ \{P\}$$



$$P \equiv I \wedge R \wedge w = \exp(X_0, z - 1)$$

#### Invariant preservation

$$\{P \wedge B\} \ C \ \{P\}$$

A second assignment axiom

$$\{I \land 0 \le z \le y \land w * x = \exp(X_0, z)\}$$
 w := w\*x  $\{I \land 0 \le z \le y \land w = \exp(X_0, z)\}$ 

Simplifying and strengthening the precondition we get:

$$\{I \land 1 \le z \le y \land w = \exp(X_0, z - 1)\}\$$
w := w\*x  $\{I \land z \ge 0 \land w = \exp(X_0, z)\}$ 



$$P \equiv I \wedge R \wedge w = \exp(X_0, z - 1)$$

Thus

$$\frac{}{\{P \wedge B\} \text{ w } := \text{ w*x } \{I \wedge z \ge 0 \wedge w = \exp(X_0, z)\}} (A_2)$$

And these can now be sequenced

$$\frac{A_2}{\{P \wedge B\}\ C\ \{P\}};$$
 
$$\{P\} \ \text{while B do C} \ \{P \wedge \neg B\}$$



#### Similarly for the initializations, going backwards

$$\frac{1}{\{I \land 1 \le z \le y + 1 \land 1 = \exp(X_0, z - 1)\} \text{ w } := 1 \text{ } \{P\}}$$

$$\frac{}{\{I\} \ \mathbf{z} \ := \ \mathbf{1} \ \{I \land 1 \le z \le y + 1 \land 1 = \exp(X_0, z - 1)\}} (A_4)$$

$$A_4$$
  $A_3$   $\{I\}$  z := 1; w := 1  $\{P\}$ 



#### Sequencing:

$$\frac{A_2}{A_4} = \frac{A_1}{\{P \land B\} \ C \ \{P\}}; \qquad \frac{A_2}{\{P \land B\} \ C \ \{P\}}; \qquad \frac{\{I\} \ \mathtt{z} \ := \ 1; \ \mathtt{w} \ := \ 1 \ \{P\} \ \mathtt{while} \ \mathtt{B} \ \mathtt{do} \ \mathtt{C} \ \{P \land \neg B\}}$$

#### and the postcondition can be weakened:

$$P \wedge \neg B \iff I \wedge R \wedge w = \exp(X_0, z - 1) \wedge \neg B$$
  
 $\iff I \wedge w = \exp(X_0, z - 1) \wedge z = y + 1$   
 $\implies I \wedge w = \exp(X_0, y)$   
 $\implies I \wedge w = \exp(X_0, Y_0)$ 



# Dealing with Arrays

Arrays can be treated as families of variables indexed by integers.

Naive axiom:

$$\{Q[a_i \mapsto e]\} a_i := e\{Q\}$$

What's wrong?



# Dealing with Arrays

The solution is to substitute arrays monolithically

$${Q[a \mapsto a^{(i,e)}]} a_i := e {Q}$$

$$a_k^{(i,e)} = \begin{cases} a_k & \text{for } k \neq i \\ e & \text{for } k = i \end{cases}$$



#### Procedures / Functions

- Introduce functional component in the language (ALGOL-style)
- Allows for recursive definitions and an additional source of non-termination
- Two classes of identifiers: assignable variables and abstraction variables
- Quantifiers can be formalized with lambdas



### Interference

```
f(x) {
  return x+k;
}
```

$${a = f(b)} k := k+1 {a = f(b)}$$



#### **Pointers**

Classic problems...

$${*q = x} *p := *p+1 {*q = x}$$



#### **Total Correctness**

The identification of a decreasing variant expression is necessary to gurantee that every loop terminates

$$\frac{[I \&\& B \&\& V == n] C [I \&\& V < n] \qquad I \&\& B \to V >= 0}{[I] \text{ while } (B) \text{ do } C [I \&\& \neg B]}$$





# Realistic Languages

The problems that need to be addressed seem daunting, however:

- -all have been studied at the theoretical level (beyond our scope)
- -most importantly, tools exist that support full languages (including object-oriented features)



#### Exercise I

```
void swap(int X[], int a, int b)
{ aux = X[a]; X[a] = X[b]; X[b] = aux; }
```

- I. Write specification
- 2. Prove correctness of function



#### Exercise 2

Recall the *partition* function used by the quicksort algorithm. Verify informally:

- I. Write a Specification
- 2. Examine suggested implementation
- 3. Identify loop invariant
- 4. Check initial conditions and presevation
- 5. Identify loop variant
- 6. Check final conditions



```
int partition (int A[], int p, int r)
 x = A[r];
 i = p-1;
 for (j=p ; j<r ; j++)
   if (A[j] <= x) {
      i++;
      swap(A, i, j);
 swap(A, i+1, r);
 return i+1;
```

#### Análise de Correcção - Invariante

No início de cada iteração do ciclo for tem-se para qualquer posição k do vector:

- 1. Se  $p \leq k \leq i$  então  $A[k] \leq x$ ;
- 2. Se  $i + 1 \le k \le j 1$  então A[k] > x;
- 3. Se k = r então A[k] = x.

- $\Rightarrow$  Verificar as propriedades de *inicialização*  $(j=p,\ i=p-1)$ , preservação, e terminação (j=r)
  - ⇒ o que fazem as duas últimas instruções?

#### Algorithms slide

#### Jump Forward



# Something Missing!

- It is still required to check that the elements are the same in the input and in the output arrays!
- A particular case of the problem of specifying that two arrays contain the same elements
- And same number of occurences: multiset equality, rather than set equality



#### A first attempt

$$\forall k : p \le k \le r : (\exists l : p \le l \le r : A[k] = B[l] \land A[l] = B[k])$$

What's wrong with it?



#### Second attempt

$$\forall k : p \le k \le r : (\exists l : p \le l \le r : A[k] = B[l])$$

$$\land$$

$$\forall k : p \le k \le r : (\exists l : p \le l \le r : B[k] = A[l])$$

What's wrong with it?



#### Third attempt

Use a logical theory for multi-sets and a function mset that abstracts an array into the multiset of its elements

$$mset(A) = mset(B)$$

This requires a prover with support for theories like sets, multisets, sequences... or else user-defined theories



# Program Annotation and Automated Static Checking





# Why Annotate Programs?

- A practical and accessible interface specification method
- Specify the semantics together with the syntax
- Do not worry about following a prescribed design method,
   as is the case with most formal methodologies
- "Light" formal methods for everyday programmers?





# **Applications**

- Dynamic checking
- Test-case generation
- Static Checking
- Documentation: register design decisions and implementation steps
- Design by Contract





# Design by Contract

- A software development method, initiated with Eiffel, based on contracts between clients and classes (dynamically-checked)
- Client guarantees certain (pre-)conditions before invoking methods and may then assume other (post-)conditions after invocation





# Design by Contract

- Class must ensure certain (post-)conditions hold after methods have been called and may for this effect assume given (pre-)conditions
- Advantages: reasoning/modularity; blame assignment;
   eliminate defensive checking (practical and efficient!!!)



# JML (Java Modelling Language)

- A standard annotation language for JML
- Is itself very close to Java (easy to learn)
- Many tools have adhered to the standard and are now
   JML-compliant
- Imperative subset has been adapted to other languages (C)



# JML Assertions

- preconditions: keyword requires
- postconditions: keyword ensures
- (class and loop) invariants:

keywords invariant

and loop invariant



# JML Assertions

Added as special comments in Java files

```
/*@ ... @*/
//@ ...
```

- Properties written as Java boolean expressions
- With extra operators...



# JML Operators

• Quantification:

```
(\forall ...; ...; ...)
(\exists ...; ...; ...)
```

- variable value at entry: \old(...)
- method return value: \result



#### Class Invariants

- Universal properties of class and instance variables (valid all the time)
- Must be preserved by all the methods in a class
- Implicitly, it is as if they were part of every preand postcondition



# Other JML Stuff

- exceptions (keyword signals)
- frame conditions
- pure methods: pure
- non\_null annotations
- ad hoc assertions: \assert



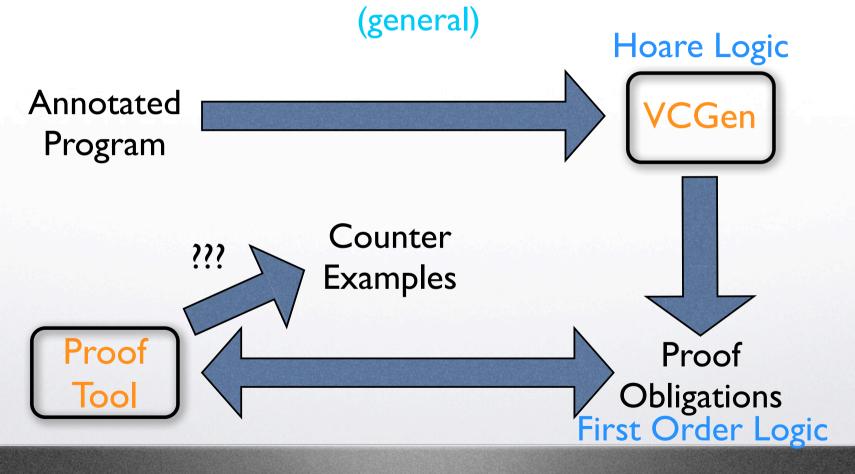
# Static Checking

- Dynamic checking verifies only the execution paths followed in one run of the program
- Static checking examines all possible execution paths
- The location of the warnings that are issued is not where they occur (as in run-time) but where they are *created*
- Typically unsound and incomplete to increase costeffectiveness (automatic theorem prover, not interactive)





# Underlying Architecture





# ESC/Java and ESC/Java2

- Development Story: DEC / Compaq / HP research labs
- ESC/Java2: Kodak and UC Dublin researchers
   (update to cover full JML and recent versions of Java)
- JML-based; attempts to check consistency of code with annotations automatically
- Current versions use the **Simplify** theorem prover





# ESC/Java and ESC/Java2

- Typical successful checks: null dereferencing; out-of-bounds array indexes (run-time exceptions).

  Safety checking
- Annotations may both suppress warnings (pre-condition prevents warning) and generate new warnings (preconditions may possibly not be met)

#### Jump back



## **DEMO**

ESC/Java2 eclipse plugin

swap / partition example



# Limitations Highlighted by Partition Example!

$$\forall k : p \le k \le r : (\exists l : p \le l \le r : A[k] = B[l] \land A[l] = B[k])$$

What's wrong with it?
Too Strong! However, ESC/Java proves this
(an example of unsoundness)



#### Second attempt

$$\forall k : p \leq k \leq r : (\exists l : p \leq l \leq r : A[k] = B[l])$$

$$\land$$

$$\forall k : p \leq k \leq r : (\exists l : p \leq l \leq r : B[k] = A[l])$$

What's wrong with it?
Too weak! However, ESC/Java fails to prove it
(an example of incompleteness)