Measuring Inconsistency in Knowledge Bases

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Seminar Presentation

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- It is shown that this measure accomplishes also an evaluation of the "intrinsic" inconsistency of a knowledge base;

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- For a formula θ in which $x \in \mathcal{V}$ occurs free, and a constant $d \in D$, define the **substitution** $\theta[x/d]$ **of** x **by** d **in** θ as the formula obtained by replacing all occurrences of the free variable x by d;

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- The well known Galois connection yields

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- Moreover, both M_1 and M_2 are *minimal* QC models;

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- Recalling the finiteness hypothesis, we define the **measure of inconsistency** for a model M in the context of $\mathcal{L}$ and D (i.e., $\text{Cnfl}(M) \subseteq \text{GrdAt}(\mathcal{L}, D)$) by

$$\boxed{\text{ModInc}(M, \mathcal{L}, D) := \frac{|\text{Cnfl}(M)|}{|\text{GrdAt}(\mathcal{L}, D)|};}$$

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Theorem (Minimal Models for Universal Formulas)

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- Thus, even if there exist many different preferred QC models of Δ, all such models have identical inconsistency measures;

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  - Thus, for all $M \in \mathrm{PQC}(\mathcal{L}, \Delta_1, D)$, $\mathrm{ModInc}(M, \mathcal{L}, D) = \frac{1}{8}$ and, for all $M \in \mathrm{PQC}(\mathcal{L}, \Delta_2, D)$, $\mathrm{ModInc}(M, \mathcal{L}, D) = \frac{1}{8}$;

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Given $\mathcal{L}$ and Δ, if D_1 and D_2 are two domains of the same size and $M_1 \in \mathrm{PQC}(\mathcal{L}, \Delta, D_1)$, $M_2 \in \mathrm{PQC}(\mathcal{L}, \Delta, D_2)$, then

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  where, for all $n \geq 1$, $r_n = \mathrm{ModInc}(M, \mathcal{L}, D_n)$, where D_n is a domain of size n and $M \in \mathrm{PQC}(\mathcal{L}, \Delta, D_n)$, if such a model exists, and $r_n = *$ (null value), otherwise;

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- The intention is to measure how the inconsistency of Δ in $\mathcal{L}$ evolves as the size of the domain increases;

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If $\Delta_1 \subseteq \Delta_2$, then $\text{ThInc}(\Delta_1, \mathcal{L}) \preceq \text{ThInc}(\Delta_2, \mathcal{L})$.

Some Comparisons

- In case $\mathcal{L}_1 = \mathcal{L}_2$, we say that Δ_1 **has smaller than or equal inconsistency as** Δ_2 and write $\Delta_1 \leq^{\mathcal{L}}_{\text{inc}} \Delta_2$;
- For fixed Δ and $\mathcal{L}$, it is not necessarily the case that $\text{ThInc}(\Delta, \mathcal{L}) = \langle r_1, r_2, \ldots \rangle$ is such that, for all i, if $r_i \neq *$, then $r_i \geq r_{i+1}$;
  Example: Consider, for instance, $\mathcal{L} = \langle \{\}, \{P(2), Q(1)\} \rangle$ and $\Delta = \{\forall x \forall y (P(x, y) \wedge \neg P(x, y))\}$; Then $\text{ThInc}(\Delta, \mathcal{L}) = \langle \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots \rangle$;

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Proposition

If $\mathcal{L}_1 = \langle \mathcal{C}_1, \mathcal{P}_1 \rangle$ and $\mathcal{L}_2 = \langle \mathcal{C}_2, \mathcal{P}_2 \rangle$, and $\mathcal{L}_1 \subseteq \mathcal{L}_2$ (meaning $\mathcal{C}_1 \subseteq \mathcal{C}_2$ and $\mathcal{P}_1 \subseteq \mathcal{P}_2$), then $\text{ThInc}(\Delta, \mathcal{L}_2) \preceq \text{ThInc}(\Delta, \mathcal{L}_1)$.

More Comparisons

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- If $\Delta_1 \subseteq \Delta_2$ and $\mathcal{L}_1 \subseteq \mathcal{L}_2$, it does not necessarily follow that $\mathsf{ThInc}(\Delta_1, \mathcal{L}_1) \preceq \mathsf{ThInc}(\Delta_2, \mathcal{L}_2)$;

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  - Then

$$\mathrm{ThInc}(\Delta_1, \mathcal{L}_1) \preceq \mathrm{ThInc}(\Delta_2, \mathcal{L}_2)$$

  - However,

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  - The theory $\Delta_3 = \{\forall x \exists y (P(x, y) \wedge \neg P(x, y))\}$ has 9 preferred QC models; One is $M_{31} = \{P(a, a), \neg P(a, a), P(b, c), \neg P(b, c), P(c, a), \neg P(c, a)\}$;

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- Thus $r_n = \mathrm{ModInc}(M, \mathcal{L}, D_n) = \frac{r}{ns}$;

Many Thanks!

MANY THANKS for

- the hospitality
- your attention during the seminar!