Process-oriented architectural design
(A crash course on interactive processes)

Luís S. Barbosa

HASLab - INESC TEC
Universidade do Minho
Braga, Portugal

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Software architecture for reactive systems

Recalling the course aims:

- Study architectural disciplines for reactive systems (often complex, time critical, mobile, etc ...)
- in which continued interaction, in different forms, emerges as a first-class citizen and the main form of composition

Syllabus

- Introduction to software architecture
- Component-oriented architectural design
    - Paradigm: Software components as monadic Mealy machines
    - Foundations: Coalgebra theory as a semantic framework for state-based systems a component calculus
    - Method: The MMM calculus; prototyping in HASKELL
- Process-oriented architectural design
    - Paradigm: Overview of process-oriented ADLs
    - Foundations: Interactive Markov chains
    - Method: Specification and analysis of architectures with stochastic constrains
- Coordination-oriented architectural design
    - Paradigm: The REO exogenous coordination model
    - Foundations: Constraint automata and variants
    - Method: Compositional specification of the glue layer

Process-oriented architectural design

- ... the oldest paradigm in SA
- several methodologies/languages/platforms available: Wright, Drawin, Acme, AADL, ...
- one under development at HASLab: ARCHERY (Alejandro Sanchez PhD thesis, 2013)

Process-oriented architectural design

Ex: a client-server configuration in ACME

```
System CS = {
   component client = { port call }
   component server = { port request }
      property max-clients-supported = 10;
   connector rpc = { role plug-cl; role plug-sv}
   }
   attachments = {
      { call to plug-cl ; server to plug-sv }
   }
```

Labelled Transition Space

Definition

A labelled transition space over a set N of names is a tuple $\langle S, N, \longrightarrow \rangle$ where

- $S = \{s_0, s_1, s_2, ...\}$ is a set of states
- $\longrightarrow \subseteq S \times N \times S$ is the transition relation, often given as an N-indexed family of binary relations

$$s \xrightarrow{a} s' \;\Leftrightarrow\; \langle s', a, s \rangle \in \longrightarrow$$

Labelled Transition Space

Morphism

A morphism relating two labelled transition spaces over N, $\langle S, N, \longrightarrow \rangle$ and $\langle S', N, \longrightarrow' \rangle$, is a function $h : S \longrightarrow S'$ st

$$s \xrightarrow{a} s' \quad \Rightarrow \quad h\ s \xrightarrow{a}{}' \ h\ s'$$

$$\boxed{\text{morphisms preserve transitions}}$$

Reachability

Definition

The reachability relation, $\longrightarrow^* \subseteq S \times N \times S$, is defined inductively

- $s \xrightarrow{\epsilon}^* s'$ for each $s \in S$, where $\epsilon \in N^*$ denotes the empty word;
- if $s \xrightarrow{\sigma}^* s''$ and $s'' \xrightarrow{a} s'$ then $s \xrightarrow{\sigma a}^* s'$, for $a \in N, \sigma \in N^*$

Reachable state

$t \in S$ is reachable from $s \in S$ iff there is a word $\sigma \in N^*$ st $s \xrightarrow{\sigma}^* t$

Labelled Transition System

Labelled Transition System

Given a labelled transition space $\langle S, N, \downarrow, \longrightarrow\rangle$, each state $s \in S$ determines a labelled transition system (LTS) over all states reachable from s and the corresponding restrictions of $\longrightarrow$.

LTS classification

- deterministic
- non deterministic
- finite
- image finite
- ...

New LTS from old

Product

$$\frac{p \xrightarrow{a} p'}{p \mid_K q \xrightarrow{a} p' \mid_K q}\ a \notin K \qquad \frac{q \xrightarrow{a} q'}{p \mid_K q \xrightarrow{a} p \mid_K q'}\ a \notin K$$

$$\frac{p \xrightarrow{a} p' \quad q \xrightarrow{a} q'}{p \mid_K q \xrightarrow{a} p' \mid_K q'}\ a \in K$$

- synchronous, multiparty interaction
- ... other interaction disciplines are possible

New LTS from old

Abstraction

$$\frac{p \xrightarrow{a} p'}{\mathrm{hide}_K\ p \xrightarrow{a} \mathrm{hide}_K\ p}\ a \notin K \qquad \frac{p \xrightarrow{a} p'}{\mathrm{hide}_K\ p \xrightarrow{\tau} \mathrm{hide}_K\ p}\ a \in K$$

- τ represents the unobservable, internal action
- product + abstraction = composition

Trace equivalence

Trace (from language theory)

A word $\sigma \in N^*$ is a trace of a state $s \in S$ iff there is another state $t \in S$ such that $s \xrightarrow{\sigma}^* t$

Trace equivalence

- Two states are trace equivalent if they have the same set of traces
- Two systems are trace equivalent if their initial states are.

Automata

Back to old friends?

$$\boxed{\text{automaton behaviour} \quad \Leftrightarrow \quad \text{accepted language}}$$

Recall that finite automata recognize regular languages, i.e. generated by

- $L_1 + L_2 \triangleq L_1 \cup L_2$ (union)
- $L_1 \cdot L_2 \triangleq \{st \mid s \in L_1, t \in L_2\}$ (concatenation)
- $L^* \triangleq \{\epsilon\} \cup L \cup (L \cdot L) \cup (L \cdot L \cdot L) \cup \ldots$ (iteration)

Automata

There is a syntax to specify such languages:

$$E \quad ::= \quad \epsilon \mid a \mid E + E \mid E\,E \mid E^*$$

where $a \in \Sigma$.

- which regular expression specifies $\{a, bc\}$?
- and $\{ca, cb\}$?

and an algebra of regular expressions:

$$\begin{aligned} (E_1 + E_2) + E_3 &= E_1 + (E_2 + E_3) \\ (E_1 + E_2)\,E_3 &= E_1\,E_3 + E_2\,E_3 \\ E_1\,(E_2\,E_1)^* &= (E_1\,E_2)^*\,E_1 \end{aligned}$$

After thoughts

... need more general models and theories
(but maybe along similar lines):

- Several interaction points ($\neq$ functions)
- Non determinisim should be taken seriously: the notion of equivalence based on accepted language is blind wrt non determinism
- Moreover: the reactive character of systems entail that not only the generated language is important, but also the states traversed during an execution of the automata.

Simulation

the quest for a behavioural equality:
able to identify states that cannot be distinguished by any realistic form of observation

Simulation

A state q simulates another state p if every transition from q is corresponded by a transition from p and this capacity is kept along the whole life of the system to which state space q belongs to.

Simulation

Definition

Given $\langle S_1, N, \longrightarrow_1\rangle$ and $\langle S_2, N, \longrightarrow_2\rangle$ over N, relation $R \subseteq S_1 \times S_2$ is a simulation iff, for all $\langle p, q\rangle \in R$ and $a \in N$,

$$p \xrightarrow{a}_1 p' \;\Rightarrow\; \langle \exists\, q' \;:\; q' \in S_2 :\; q \xrightarrow{a}_2 q' \wedge \langle p', q'\rangle \in R\rangle$$

$$\begin{array}{ccc} \begin{array}{ccc} p & R & q \\ \downarrow{\scriptstyle a} & & \\ p' & & \end{array} & \Rightarrow & \begin{array}{ccc} & & q \\ & & \downarrow{\scriptstyle a} \\ p' & R & q' \end{array} \end{array}$$

Example

$$
\begin{tikzcd}
 & q_1 \arrow[r, "d"] & q_2 \\
q_0 \arrow[ur, "a"] \arrow[dr, "a"'] & & \\
 & q_4 \arrow[r, "e"'] & q_3
\end{tikzcd}
\qquad
\begin{tikzcd}
 & & p_2 \\
p_0 \arrow[r, "a"] & p_1 \arrow[ur, "d"] \arrow[dr, "e"'] & \\
 & & p_3
\end{tikzcd}
$$

$$q_0 \lesssim p_0 \qquad \text{cf.} \qquad \{\langle q_0, p_0\rangle, \langle q_1, p_1\rangle, \langle q_4, p_1\rangle, \langle q_2, p_2\rangle, \langle q_3, p_3\rangle\}$$

Similarity

Definition

$$p \lesssim q \;\Leftrightarrow\; \langle \exists\, R \;::\; R \text{ is a simulation and } \langle p, q\rangle \in R\rangle$$

Lemma

The similarity relation is a preorder
(ie, reflexive and transitive)

Bisimulation

Definition

Given $\langle S_1, N, \longrightarrow_1\rangle$ and $\langle S_2, N, \longrightarrow_2\rangle$ over N, relation $R \subseteq S_1 \times S_2$ is a bisimulation iff both R and its converse R° are simulations.
I.e., whenever $\langle p, q\rangle \in R$ and $a \in N$,

$$
\begin{array}{ll}
(1) & p \xrightarrow{a}_1 p' \;\Rightarrow\; \langle \exists\, q' \;:\; q' \in S_2 :\; q \xrightarrow{a}_2 q' \wedge \langle p', q'\rangle \in R\rangle \\
(2) & q \xrightarrow{a}_2 q' \;\Rightarrow\; \langle \exists\, p' \;:\; p' \in S_1 :\; p \xrightarrow{a}_1 p' \wedge \langle p', q'\rangle \in R\rangle
\end{array}
$$

Examples

$$\begin{tikzcd}
 & q_1 \arrow[dl, "a"'] \arrow[dr, "a"] & \\
q_2 \arrow[rr, "c"] & & q_3 \arrow[loop right, "c"]
\end{tikzcd}
\qquad\qquad
\begin{tikzcd}
m \arrow[d, "a"'] \\
n \arrow[loop right, "c"]
\end{tikzcd}$$

$$\begin{tikzcd}
q_1 \arrow[r, "a"] & q_2 \arrow[r, "a"] & q_3 \arrow[r, "a"] & \cdots
\end{tikzcd}
\qquad\qquad
\begin{tikzcd}
h \arrow[loop right, "a"]
\end{tikzcd}$$

Bisimilarity

Definition

$$p \sim q \;\Leftrightarrow\; \langle \exists\, R \;::\; R \text{ is a bisimulation and } \langle p, q \rangle \in R \rangle$$

Lemma

1. The identity relation id is a bisimulation
2. The empty relation $\perp$ is a bisimulation
3. The converse R° of a bisimulation is a bisimulation
4. The composition $S \cdot R$ of two bisimulations S and R is a bisimulation
5. The $\bigcup_{i \in I} R_i$ of a family of bisimulations $\{R_i \mid i \in I\}$ is a bisimulation

Bisimilarity

Lemma

The bisimilarity relation is an equivalence relation
(ie, reflexive, symmetric and transitive)

Lemma

The class of all bisimulations between two LTS has the structure of a complete lattice, ordered by set inclusion, whose top is the bisimilarity relation $\sim$.

Bisimulation

Definition (alternative)

Given $\langle S_1, N, \longrightarrow_1\rangle$ and $\langle S_2, N, \longrightarrow_2\rangle$ over N, relation $R \subseteq S_1 \times S_2$ is a bisimulation iff

$$\langle p, q\rangle \in R \;\Leftrightarrow\; \langle \forall\, a, C \;:\; a \in N, C \in (S_1 \cup S_2)/R :\; p \xrightarrow{a}_1 C \Leftrightarrow q \xrightarrow{a}_2 C\rangle$$

where, for an equivalence class C,

$$p \xrightarrow{a} C \;\Leftrightarrow\; \langle \exists\, p' \;:\; p' \in C :\; p \xrightarrow{a} p'\rangle$$

Bisimilarity

Warning

$\boxed{\text{The bisimilarity relation } \sim \text{ is not the symmetric closure of } \lesssim}$

Example

$$q_0 \lesssim p_0, \; p_0 \lesssim q_0 \quad \text{but} \quad p_0 \not\sim q_0$$

$$
\begin{tikzcd}
 & q_1 & \\
q_0 \arrow[ur, "a"] \arrow[dr, "a"'] & & \\
 & q_2 \arrow[r, "b"] & q_3
\end{tikzcd}
\qquad\qquad
\begin{tikzcd}
p_0 \arrow[r, "a"] & p_1 \arrow[r, "b"] & p_3
\end{tikzcd}
$$

Notes

Similarity as the greatest simulation

$$\lesssim \ \triangleq \ \bigcup\{S \mid S \text{ is a simulation}\}$$

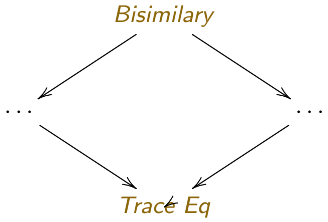
Bisimilarity as the greatest bisimulation

$$\sim \ \triangleq \ \bigcup\{S \mid S \text{ is a bisimulation}\}$$

cf relational translation of definitions
$\lesssim$ and $\sim$ as greatest fix points (Tarski's theorem)

Notes

The Van Glabbeek linear - branching time spectrum



Complexity

- Virtually all forms of bisimulation can be determined in polynomial time on finite state transition systems
- ... whereas trace, or language equivalence are in general difficult (P-space hard)

Abstraction

Main idea:
Take a set of actions as internal or non-observable

Approaches

- R. Milner's weak bisimulation [Mil80]
- Van Glabbeek and Weijland's branching bisimulation [GW96]

Abstraction

- Intuition similar to that of strong bisimulation: But now, instead of letting a single action be simulated by a single action, within an envelope of internal transitions
- An internal action τ can be simulated by any number of internal transitions (even by none).

Weak bisimulation

Definition [Milner,80]

Given $\langle S_1, N, \longrightarrow_1\rangle$ and $\langle S_2, N, \longrightarrow_2\rangle$ over N, relation $R \subseteq S_1 \times S_2$ is a weak bisimulation iff for all $\langle p, q\rangle \in R$ and $a \in N$,

1. If $p \xrightarrow{a}_1 p'$, then
   - either $a = \tau$ and $p' R q$
   - or, there is a sequence $q \xrightarrow{\tau}_2 \cdots \xrightarrow{\tau}_2 t \xrightarrow{a}_2 t' \xrightarrow{\tau}_2 \cdots \xrightarrow{\tau}_2 q'$ involving zero or more τ-transitions, such that $p' R q'$.
2. symmetrically ...

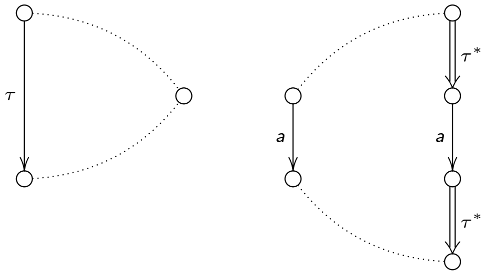
Note

it corresponds to a strong bisimulation over $\stackrel{s}{\Longrightarrow}$ for $s \in N^*$

Weak bisimilarity

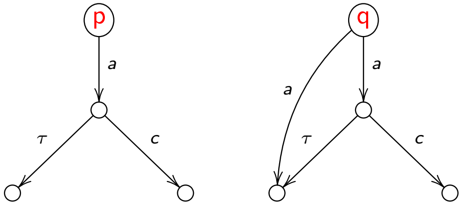
Definition

$$p \approx_w q \;\Leftrightarrow\; \langle \exists\ R \;::\; R \text{ is a branching bisimulation and } \langle p, q\rangle \in R \rangle$$



Example

abstracts over intern action but branching is not preserved



Branching bisimulation

Definition

Given $\langle S_1, N, \longrightarrow_1\rangle$ and $\langle S_2, N, \longrightarrow_2\rangle$ over N, relation $R \subseteq S_1 \times S_2$ is a branching bisimulation iff for all $\langle p, q\rangle \in R$ and $a \in N$,

1. If $p \xrightarrow{a}_1 p'$, then
   - either $a = \tau$ and $p' R q$
   - or, there is a sequence $q \xrightarrow{\tau}_2 \cdots \xrightarrow{\tau}_2 q'$ of (zero or more) τ-transitions such that pRq' and $q' \xrightarrow{a}_2 q''$ with $p' R q''$.

   pRq' and $q' \downarrow_2$.
2. symmetrically ...

Exercise

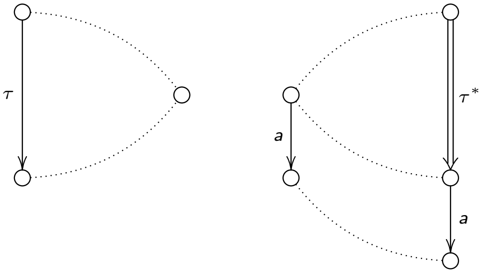
Give an alternative definition in terms of equivalence classes

Branching bisimilarity

Definition

$$p \approx_b q \;\Leftrightarrow\; \langle \exists\ R \ :: \ R \text{ is a branching bisimulation and } \langle p, q \rangle \in R \rangle$$

... preserves the branching structure



Divergence

Branching and weak bisimilarity do not preserve τ-loops

satisfying a notion of fairness: if a τ-loop exists, then no infinite execution sequence will remain in it forever if there is a possibility to leave

Exercise

Modify the corresponding definitions to enforce preserving divergence

The rootedness condition

Problem

If an alternative is added to the initial state then transition systems that were branching bisimilar may cease to be so.

Example: add a b-labelled branch to the initial states of

Rooted branching bisimilarity

Startegy

Impose a rootedness condition [R. Milner, 80]:

Initial τ-transitions can never be inert, *i.e.*, two states are equivalent if they can simulate each other's initial transitions, such that the resulting states are branching bisimilar.

Rooted branching bisimulation

Definition

Given $\langle S_1, N, \longrightarrow_1 \rangle$ and $\langle S_2, N, \longrightarrow_2 \rangle$ over N, relation $R \subseteq S_1 \times S_2$ is a rooted branching bisimulation iff

1. it is a branching bisimulation
2. for all $\langle p, q \rangle \in R$ and $a \in N$,
   - If $p \xrightarrow{a}_1 p'$, then there is a $q' \in S_2$ such that $q \xrightarrow{a}_2 q'$ and $p' \approx_b q'$
   - If $q \xrightarrow{a}_2 q'$, then there is a $p' \in S_1$ such that $p \xrightarrow{a}_1 p'$ and $p' \approx_b q'$

Rooted branching bisimilarity

Definition

$p \approx_{rb} q \;\Leftrightarrow\; \langle \exists\, R \;::\; R$ is a rooted branching bisimulation and $\langle p, q \rangle \in R \rangle$

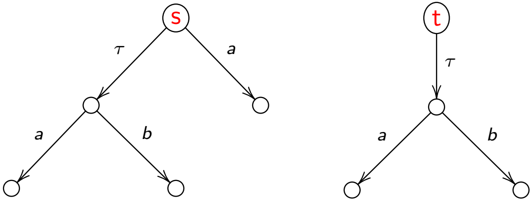
Lemma

$$\sim \;\subseteq\; \approx_{rb} \;\subseteq\; \approx_b$$

Of course, in the absence of τ actions, $\sim$ and $\approx_b$ coincide.

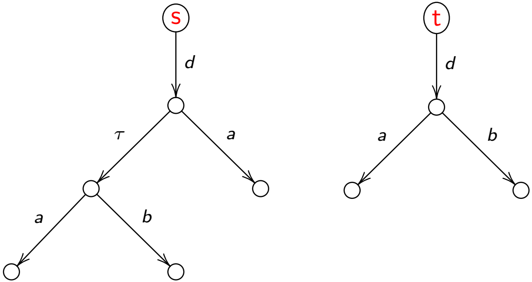
Example

branching but not rooted



Example

rooted branching bisimilar



Rooted weak bisimilarity

The same recipe applies to weak bisimilarity:

Definition

$$p \approx_{rw} q \;\Leftrightarrow\; \langle \exists\, R \;::\; R \text{ is a rooted weak bisimulation and } \langle p, q\rangle \in R \rangle$$

Lemma

$$\begin{tikzcd}
 & \approx_w \arrow[dl, dash] \arrow[dr, dash] & \\
\approx_b \arrow[dr, dash] & & \approx_{rw} \arrow[dl, dash] \\
 & \approx_{rb} \arrow[d, dash] & \\
 & \sim &
\end{tikzcd}$$

(ordered by $\subseteq$)

The questions to follow ...

- We already have a semantic model for reactive systems. With which language shall we describe them?
- How to compare and transform such systems?
- How to express and prove their proprieties?

$\rightsquigarrow$ process languages and calculi
cf. CCS (Milner, 80), CSP (Hoare, 85),
ACP (Bergstra & Klop, 82),
π-*calculus* (Milner, 89), among many others

$\rightsquigarrow$ modal (temporal, hybrid) logics

Process algebra

$\boxed{\text{mCRL2 as a prototypical (source of) process algebras}}$

Modal logics

$\boxed{\text{The Hennessy-Milner logic and modal equivalence}}$

Process-oriented architectural design

Module project

- Explore AADL (namely its BA - behavioural annex) providing a hands-on comparison with coordination-oriented approaches (e.g. Reo)

The lectures

- ... going a step ahead towards capturing probabilistic behaviour and composition: interactive Markov chains