UCE: MFES-10/11
CSI Module — Exercise list

M.Sc. Degrees
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NB: Equation numbers of the form ([2]:n) are taken from [2].

Exercise 1. (adapted from exercise 5.1.4 in C.B. Jones's *Systematic Software Development Using VDM* [1]):

> Hotel room numbers are pairs (l, r) where l indicates a floor and r a door number in floor l. Write the invariant on room numbers which captures the following rules valid in a particular hotel with 25 floors, 60 rooms per floor:
> 1. there is no floor number 13; (guess why)
> 2. level 1 is an open area and has no rooms;
> 3. the top five floors consist of large suites and these are numbered with even integers.

Exercise 2. Check rule

$$\langle \exists\, i \;:\; R :\; T \rangle = \langle \exists\, i \;:\; T :\; R \rangle \tag{1}$$

□

Exercise 3. Check **carefully** which rules of the quantifier calculus need to be applied to prove that predicate

$$\langle \forall\, b, a \;:\; \langle \exists\, c \;:\; b = f\ c :\; r(c, a) \rangle :\; s(b, a) \rangle \tag{2}$$

is the same as

$$\langle \forall\, c, a \;:\; r(c, a) :\; s(f\ c, a) \rangle \tag{3}$$

where f is a function and r, s are binary predicates.
□

Exercise 4. Define relations $C \xleftarrow{R} A$, $A \xleftarrow{S} B$ such that $cRa = r(c, a)$ and $bSa = s(b, a)$. Then PF-transform (2) and (3), showing that the equivalence proved above is nothing but the rule

$$f \cdot R \subseteq S \Leftrightarrow R \subseteq f^{\circ} \cdot S \tag{4}$$

which is number ([2]:67) in the tutorial. □

Exercise 5. Given a function $B \xleftarrow{f} A$, show that *img* f is the coreflexive Φ_p of predicate $p\,x \triangleq \langle \exists\, a \,::\, x = f\,a \rangle$.
□

Exercise 6. Justify the following PF calculation of ([2]:67), where the equivalence is proved by cyclic implication ("ping-pong"):

$$
\begin{array}{ll}
 & f \cdot R \subseteq S \\
\Rightarrow & \{ \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \} \\
 & f^\circ \cdot f \cdot R \subseteq f^\circ \cdot S \\
\Rightarrow & \{ \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \} \\
 & R \subseteq f^\circ \cdot S \\
\Rightarrow & \{ \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \} \\
 & f \cdot R \subseteq f \cdot f^\circ \cdot S \\
\Rightarrow & \{ \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \} \\
 & f \cdot R \subseteq S
\end{array}
$$

□

Exercise 7. So, for f entire and simple ($\Leftrightarrow$ a function) rule ([2]:67) holds. Now, suppose that rule ([2]:67) holds for f replaced by an arbitrary relation X:

$$X \cdot R \subseteq S \Leftrightarrow R \subseteq X^\circ \cdot S \tag{5}$$

Check what you can infer about this rule for the particular instantiations:

– $R, S := id, X$ (left-cancellation)
– $S, R := id, X^\circ$ (right-cancellation)

Conclude that (5) holds **if and only if** X is a function.
□

Exercise 8. Complete the following calculation about functions:

$$
\begin{array}{ll}
 & f \subseteq g \\
\Leftrightarrow & \{ \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \} \\
 & f \cdot id \subseteq g \\
\Leftrightarrow & \{ \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \} \\
 & id \subseteq f^\circ \cdot g
\end{array}
$$

$\Leftrightarrow$ { ..... }

$id \cdot g^\circ \subseteq f^\circ$

$\Leftrightarrow$ { ..... }

$g \subseteq f$

So $f \subseteq g \;\Leftrightarrow\; g \subseteq f$. Therefore

$$f \subseteq g \Leftrightarrow f = g \Leftrightarrow f \supseteq g \tag{6}$$

Why?
□

Exercise 9. Recalling universal properties (Galois connections)

$$X \subseteq R \wedge X \subseteq S \;\Leftrightarrow\; X \subseteq R \cap S \tag{7}$$
$$R \cup S \subseteq X \;\Leftrightarrow\; R \subseteq X \wedge S \subseteq X \tag{8}$$
$$X \cdot R \subseteq Y \;\Leftrightarrow\; X \subseteq Y \,/\, R \tag{9}$$

resort to the *indirect equality* (IE) rule to calculate the following property of relation (right) division:

$$Q \,/\, (R \cup S) = (Q \,/\, R) \cap (Q \,/\, S) \tag{10}$$

Moreover, resort to

$$\langle \forall\, b \;:\; a\; R\; b :\; c\; S\; b \rangle \qquad \begin{array}{ccc} C & \xleftarrow{S/R} & A \\ & {\scriptstyle S}\nwarrow \;\supseteq\; \nearrow{\scriptstyle R} & \\ & B & \end{array} \tag{11}$$

in converting (10) to PW-notation. Which rule of universal quantification have you calculated?

Exercise 10. Resort to

$$\Phi_q \xleftarrow{R} \Phi_p \quad \Leftrightarrow \quad R \cdot \Phi_p \;\subseteq\; \Phi_q \cdot \top \tag{12}$$

in calculating the **split by conjunction** rule of the PO calculus of [2]:

$$\Phi_{q_1} \cdot \Phi_{q_2} \xleftarrow{R} \Phi_p \quad \Leftrightarrow \quad \Phi_{q_1} \xleftarrow{R} \Phi_p \;\wedge\; \Phi_{q_2} \xleftarrow{R} \Phi_p \tag{13}$$

NB: you will need the following distribution property,

$$(\Phi \cap \Psi) \cdot \top = (\Phi \cdot \top) \cap (\Psi \cdot \top) \tag{14}$$

easy to prove using indirect equality and GC ($f = \rho, g = (\cdot \top)$) — do it.

Exercise 11. Given A e B, build their *disjoint union* by defining $A + B \triangleq \{i_1\ a \mid a \in A\} \cup \{i_2\ b \mid b \in B\}$, where $i_1\ a \triangleq (1, a)$ and $i_2\ b \triangleq (2, b)$ are the *injections* associated to this construction, which are mutually orthogonal:

$$i_2^\circ \cdot i_1 = \bot \tag{15}$$

Over this construction build the "junc" (ou "either") relational combinator, as follows:

$$[R\ , S] \triangleq R \cdot i_1^\circ \cup S \cdot i_2^\circ \tag{16}$$

It can be shown that (16) enjoys *universal* property

$$[R\ , S] \subseteq X \quad \Leftrightarrow \quad R \subseteq X \cdot i_1 \ \wedge \ S \subseteq X \cdot i_2 \tag{17}$$

which strengthens to equality:

$$X = [R\ , S] \quad \Leftrightarrow \quad X \cdot i_1 = R \ \wedge \ X \cdot i_2 = S \tag{18}$$

and a number of other properties, namely

$$[i_1\ , i_2] \quad = \quad id \tag{19}$$

$$X \cdot [R\ , S] \quad = \quad [X \cdot R\ , X \cdot S] \tag{20}$$

$$[R\ , S] \subseteq [X\ , Y] \quad \Leftrightarrow \quad \begin{cases} R \subseteq X \\ S \subseteq Y \end{cases} \tag{21}$$

$$[R\ , S] \cdot [T\ , U]^\circ \quad = \quad (R \cdot T^\circ) \cup (S \cdot U^\circ) \tag{22}$$

$$[R\ , S] \cdot i_1 = R \quad \wedge \quad [R\ , S] \cdot i_2 = S \tag{23}$$

$$[R\ , S] \cdot (P + Q) \quad = \quad [R \cdot P\ , S \cdot Q] \tag{24}$$

Prove that (19), (23) and (21) hold.

Exercise 12. Consider the relational definition of the McCarthy's conditional combinator

$$p \rightarrow f, g \ \triangleq \ f \cdot \Phi_p \cup g \cdot \Phi_{\neg p} \tag{25}$$

and the definition of function $A + A \xleftarrow{p?} A$ (known as the "guard" associated to predicate p) which follows:

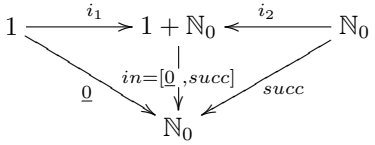
$$p? \triangleq i_1 \cdot \Phi_p \cup i_2 \cdot \Phi_{\neg p} \tag{26}$$

Show that

$$p \rightarrow f, g \quad = \quad [f\ , g] \cdot p? \tag{27}$$

holds.

Exercise 13. See below the diagram of function $in = [\underline{0}\ , succ]$ which expresses the way natural numbers are built as a Peano-algebra,



(28)

where $\underline{0}$ denotes the everywhere-0 constant function $\underline{0}\ x \triangleq 0$, and $succ\ n \triangleq n + 1$. (Type 1 is inhabited by a single, fixed element, which we don't need to denote explicitly for the moment).

1. Add justifications to the calculation below which, assuming the orthogonality condition

$$succ^\circ \cdot \underline{0} = \bot \tag{29}$$

shows that in is injective:

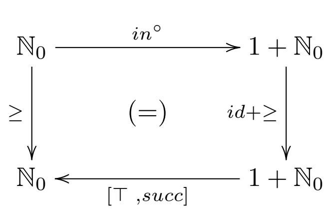
$$
\begin{array}{cl}
 & in^\circ \cdot in \subseteq id \\
\Leftrightarrow & \{ \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \} \\
 & [in^\circ \cdot \underline{0}\,, in^\circ \cdot succ] \subseteq [i_1\,, i_2] \\
\Leftrightarrow & \{ \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \} \\
 & \left\{ \begin{array}{l} in^\circ \cdot \underline{0} \subseteq i_1 \\ in^\circ \cdot succ \subseteq i_2 \end{array} \right. \\
\Leftrightarrow & \{ \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \} \\
 & \left\{ \begin{array}{l} i_1 \cdot \underline{0}^\circ \cdot \underline{0} \cup i_2 \cdot succ^\circ \cdot \underline{0} \subseteq i_1 \\ i_1 \cdot \underline{0}^\circ \cdot succ \cup i_2 \cdot succ^\circ \cdot succ \subseteq i_2 \end{array} \right. \\
\Leftrightarrow & \{ \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \} \\
 & \left\{ \begin{array}{l} i_1 \cdot \underline{0}^\circ \cdot \underline{0} \subseteq i_1 \\ i_2 \cdot succ^\circ \cdot \underline{0} \subseteq i_1 \\ i_1 \cdot \underline{0}^\circ \cdot succ \subseteq i_2 \\ i_2 \cdot succ^\circ \cdot succ \subseteq i_2 \end{array} \right. \\
\Leftrightarrow & \{ \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \} \\
 & \left\{ \begin{array}{l} \underline{0}^\circ \cdot \underline{0} \subseteq ker\, i_1 \\ succ^\circ \cdot \underline{0} \subseteq i_2^\circ \cdot i_1 \\ \underline{0}^\circ \cdot succ \subseteq i_1^\circ \cdot i_2 \\ succ^\circ \cdot succ \subseteq ker\, i_2 \end{array} \right. \\
\Leftrightarrow & \{ \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \} \\
 & \left\{ \begin{array}{l} \underline{0}^\circ \cdot \underline{0} \subseteq id \\ succ^\circ \cdot \underline{0} \subseteq i_2^\circ \cdot i_1 \\ succ^\circ \cdot succ \subseteq id \end{array} \right. \\
\Leftrightarrow & \{ \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \} \\
 & \left\{ \begin{array}{l} succ^\circ \cdot \underline{0} = \bot \\ succ \text{ is injective} \end{array} \right.
\end{array}
$$

2. Fact

$$img\, \underline{0} \cup img\, succ = id \tag{30}$$

records that in is a surjection. Why?

Exercise 14. The following diagram



displays the relational equality

$$\geq = [\top \, , succ] \cdot (id + \geq) \cdot in^{\circ} \tag{31}$$

which defines (inductively) the *at least* ordering on natural numbers. Show that (31) means the same as

$$n \geq m \;\; \Leftrightarrow \;\; m = 0 \;\vee\; \langle \exists\, y, x \;:\; n = y + 1 \;\wedge\; m = x + 1 :\; y \geq x \rangle \tag{32}$$

Further show that (32) could have been written as the pair of clauses

$$\begin{aligned} &\langle \forall\, n \;::\; n \geq 0 \rangle \\ &n \geq (m+1) \Leftrightarrow \langle \exists\, x \;:\; n = x + 1 :\; x \geq m \rangle \end{aligned}$$

Don't provide a pointwise argument: just rely on relation algebra results studied thus far.

Exercise 15. Let $A^{\star} \triangleq \bigcup_{i \geq 0} A^{i}$ be the set of all finite sequences on a set A, and define functions in, nil and $cons$ such that

$$\begin{aligned} in &= [nil \, , cons] \\ nil\; x &= [\,] \\ cons(a, x) &= a : x \end{aligned}$$

where $[\,]$ denotes the empty sequence and notation $a : x$ means the insertion of a at the front of sequence x. Given some relation $B \xleftarrow{R} A$, let $R^{\star}$ be the relation described in the following diagram:

$$\begin{array}{ccc} A^{\star} & \xleftarrow{\;in\;} & 1 + A \times A^{\star} \\ {\scriptstyle R^{\star}}\downarrow & (=) & \downarrow{\scriptstyle id + id \times R^{\star}} \\ B^{\star} & \xleftarrow[[nil \, , cons \cdot (R \times id)]]{} & 1 + A \times B^{\star} \end{array}$$

1. Infer from the diagram the following pointwise definition of $R^{\star}$, made of two clauses:

$$\begin{aligned} y \; R^{\star} \; [\,] &\Leftrightarrow y = [\,] \\ y \; R^{\star} \; (a : x) &\Leftrightarrow \langle \exists\, b, z \;:\; y = (b : z) :\; b \; R \; a \;\wedge\; z \; R^{\star} x \rangle \end{aligned}$$

   NB: rely on the following property of relational product

$$(R \times Q) \cdot (S \times P) = (R \cdot S) \times (Q \cdot P) \tag{33}$$

2. Check which of the following facts hold:

$$[0] \leq^{\star} [1] \tag{34}$$

$$[\,] \leq^{\star} [0] \tag{35}$$

3. Calculate $f^{\star}$. (Special case for functions.)

Exercise 16. From the free theorem of $1 \xleftarrow{!} A$ and fact $ker\,! = \top$ infer

$$f \cdot R \subseteq \top \cdot S \Leftrightarrow R \subseteq \top \cdot S \tag{36}$$

Exercise 17. Calculating with Alloy sequences, cf. eg. `sequence.als`:

```
sig Seq {
   seqElems: SeqIdx → lone elem
}
```

that is, sequences are $\mathbb{N}$ to A simple relations ($0 \notin \mathbb{N}$):

$$Seq\ A = \ \mathbb{N} \longrightarrow A$$
$$\textbf{inv}\ \ L \triangleq noHoles\ L$$

where

$$noHoles\ L \triangleq \ L \cdot succ \subseteq \top \cdot L \tag{37}$$

Operators:

$$tail\ L \triangleq \ L \cdot succ \tag{38}$$
$$head\ L \triangleq \ L \cdot img\,\underline{1} \tag{39}$$
$$c : L \triangleq \ \underline{c} \cdot \underline{1}^\circ \cup L \cdot succ^\circ \tag{40}$$

1. Transform (37) to PW-notation and check which of the following sequences represent sequence $[a, b, a]$:

$$\begin{array}{c|c} A & \mathbb{N} \\ \hline a & 2 \\ a & 3 \\ b & 1 \end{array} , \begin{array}{c|c} A & \mathbb{N} \\ \hline a & 2 \\ a & 4 \\ b & 3 \end{array} , \begin{array}{c|c} A & \mathbb{N} \\ \hline a & 3 \\ a & 1 \\ b & 2 \end{array}$$

2. Knowing that

$$img\,\underline{1} \cup img\,succ = id \tag{41}$$

   show that $L = head\ L \cup (tail\ L) \cdot succ^\circ$.
   NB: add variables to (41) beforehand just to see what it means.

Exercise 18. Show that $\Phi_{noHoles} \xleftarrow{tail} \Phi_{noHoles}$ holds, that is, $tail\ L$ preserves invariant $noHoles$, that is, complete:

$$\Phi_{noHoles} \xleftarrow{tail} \Phi_{noHoles}$$

$\Leftrightarrow$ { go pointwise ($tail$ is a function) }

$\langle \forall\ L\ :\ noHoles\ L :\ noHoles(tail\ L) \rangle$

$\Leftrightarrow$ { inline (37) ; trading (48) ; assume quantifier }

$L \cdot succ \subseteq \top \cdot L\ \Rightarrow$ .....

... { .... }

.....

Exercise 19. Complete the proof below so as to show that $\Phi_{noHoles} \xleftarrow{(c:)} \Phi_{noHoles}$ holds:

$$L \cdot succ \subseteq \top \cdot L \;\Rightarrow\; (c : L) \cdot succ \subseteq \top \cdot (c : L)$$

We show that consequent $(c : L) \cdot succ \subseteq \top \cdot (c : L)$ is entailed by antecedent $L \cdot succ \subseteq \top \cdot L$:

$$(c : L) \cdot succ \subseteq \top \cdot (c : L)$$
$\Leftrightarrow$ { .... }
$$(\underline{c} \cdot \underline{1}^\circ \cup L \cdot succ^\circ) \cdot succ \subseteq \top \cdot (c : L)$$
$\Leftrightarrow$ { .... }
$$\underline{c} \cdot \underline{1}^\circ \cdot succ \cup L \cdot succ^\circ \cdot succ \subseteq \top \cdot (c : L)$$
$\Leftrightarrow$ { .... }
$$\begin{cases} \underline{1}^\circ \cdot succ \subseteq \top \cdot (c : L) \\ L \cdot succ^\circ \cdot succ \subseteq \top \cdot (c : L) \end{cases}$$
$\Leftrightarrow$ { .... }
$$\begin{cases} \bot \subseteq \top \cdot (c : L) \\ L \cdot (img\, \underline{1} \cup img\, succ) \subseteq \top \cdot (c : L) \end{cases}$$
$\Leftrightarrow$ { .... }
$$\begin{cases} L \cdot img\, \underline{1} \subseteq \top \cdot (\underline{c} \cdot \underline{1}^\circ \cup L \cdot succ^\circ) \\ L \cdot (img\, succ) \subseteq \top \cdot (\underline{c} \cdot \underline{1}^\circ \cup L \cdot succ^\circ) \end{cases}$$
$\Leftarrow$ { .... }
$$\begin{cases} L \cdot img\, \underline{1} \subseteq \top \cdot \underline{1}^\circ \\ L \cdot (img\, succ) \subseteq \top \cdot L \cdot succ^\circ \end{cases}$$
$\Leftrightarrow$ { .... }
$$\begin{cases} L \cdot \underline{1} \cdot \subseteq \top \cdot \top \\ L \cdot (\rho\, succ) \subseteq \top \cdot L \cdot succ^\circ \end{cases}$$
$\Leftrightarrow$ { .... }
$$\begin{cases} L \cdot \underline{1} \cdot \subseteq \top \\ L \cdot (\delta\, (succ^\circ)) \subseteq \top \cdot L \cdot succ^\circ \end{cases}$$
$\Leftrightarrow$ { .... }
$$L \cdot succ \subseteq \top \cdot L$$

Exercise 20. Consider the definition of a new relation operator

$$shrink(R, S) \triangleq R \cap S/R^\circ \tag{42}$$

1. Add variables to this definition and check the following encoding of this combinator in Alloy:

```
fun shrink[r: K → A, s: A → A] : K → A {
  { a : r·dom, b : a·r | (all b' : a·r | b' in s·b) }
}
```

2. Check the outcome of $shrink(R, \leq)$ for R the relation

$\mathbb{N}$	A
10	$John$
11	$Mary$
12	$John$
15	$Arthur$

   NB: The aim of the *shrink* combinator is to convert a given relation R into a simple relation by looking at particular (eg. maximal) elements of its range relative to some ordering (eg. $\leq$).
3. Use indirect equality to show that definition (42) is equivalent to the universal property (Galois connection)

$$X \subseteq shrink(R, S) \Leftrightarrow X \subseteq R \ \wedge \ X \cdot R^{\circ} \subseteq S \tag{43}$$

4. Resort to (43) in showing that
   (a) $shrink(R, \top) = R$ for all R.
   (b) $shrink(R, id) = R$ if R is simple.

Exercise 21. Suppose you want to adapt $shrink$ so as to work over lists of pairs:

```
shrink       :: [(b,a)] -> ((b,b) -> Bool) -> [(b,a)]
```

Calculate the FT of `shrink`.

Exercise 22. Consider the definition which follows,

$$f \dot{\leq} g \triangleq f \subseteq (\leq) \cdot g \tag{44}$$

where $\leq$ is a partial order.

- Convert this definition to pointwise notation and check its meaning.
- Show that $f \dot{\leq} g$ means the same as $f(\leq \leftarrow id)g$

□

Exercise 23. Consider the following requirements for a $\mathbb{N}$ to $\mathbb{N}$ function:

> *Given a set $S \subseteq \mathbb{N}$, $\mathbb{N} \xrightarrow{reindex\ S} \mathbb{N}$ is the least function, in the sense of (44), which maps all numbers in S to an initial segment of $\mathbb{N}$.*

Consider the following specification of $reindex\ S$ (universal property): for all k, S

$$k \text{ monotone } \wedge \ k \cdot \Phi_S \text{ injective} \Leftrightarrow reindex\ S \dot{\leq} k \tag{45}$$

1. Spell out "k monotone" and "$k \cdot \Phi_S$ injective" using relational algebra notation.
2. From (45) show that, for all S, function $reindex\ S$ is a subrelation of the $\leq$ ordering on $\mathbb{N}$, that is, for all $n \in \mathbb{N}$, $(reindex\ S)n \leq n$.
3. Using an informal drawing, sketch function $reindex\{2, 3, 6\}$.
4. Show that $reindex\ \emptyset = reindex\{i\} = \underline{1}$.

Nesting:

$$\langle \forall\ a, b\ :\ R\ \wedge\ S :\ T \rangle = \langle \forall\ a\ :\ R :\ \langle \forall\ b\ :\ S :\ T \rangle \rangle \tag{46}$$

$$\langle \exists\ a, b\ :\ R\ \wedge\ S :\ T \rangle = \langle \exists\ a\ :\ R :\ \langle \exists\ b\ :\ S :\ T \rangle \rangle \tag{47}$$

Trading:

$$\langle \forall\ i\ :\ R\ \wedge\ S :\ T \rangle = \langle \forall\ i\ :\ R :\ S \Rightarrow T \rangle \tag{48}$$

$$\langle \exists\ i\ :\ R\ \wedge\ S :\ T \rangle = \langle \exists\ i\ :\ R :\ S\ \wedge\ T \rangle \tag{49}$$

Splitting:

$$\langle \forall\ j\ :\ R :\ \langle \forall\ k\ :\ S :\ T \rangle \rangle = \langle \forall\ k\ :\ \langle \exists\ j\ :\ R :\ S \rangle :\ T \rangle \tag{50}$$

$$\langle \exists\ j\ :\ R :\ \langle \exists\ k\ :\ S :\ T \rangle \rangle = \langle \exists\ k\ :\ \langle \exists\ j\ :\ R :\ S \rangle :\ T \rangle \tag{51}$$

References

1. C.B. Jones. *Systematic Software Development Using VDM*. Series in Computer Science. Prentice-Hall Int., 1990. 1st edition (1986).
2. J.N. Oliveira. *Extended Static Checking by Calculation using the Pointfree Transform* . In A. Bove et al., editor, *LerNet ALFA Summer School 2008*, volume 5520 of *LNCS*, pages 195–251. Springer-Verlag, 2009.