Exercises 1: Software Architecture for Reactive Systems

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Exercise I.1

Given $\langle S_1, \mathcal{N}, \downarrow_\infty, \longrightarrow_\infty \rangle$ and $\langle S_2, \mathcal{N}, \downarrow_\in, \longrightarrow_\in \rangle$ over $\mathcal{N}$, two states p and q are *mutually similar* iff

$$p \doteqdot q \;\Leftrightarrow\; p \lesssim q \wedge q \lesssim p$$

1. Show that $\doteqdot$ is an equivalence relation.
2. Compare this relation with bisimilarity ans language equivalence.

Exercise I.2

Consider the following transition space

$$\{\langle 1,a,2\rangle, \langle 1,a,3\rangle, \langle 2,a,3\rangle, \langle 2,b,1\rangle, \langle 3,a,3\rangle, \langle 3,b,1\rangle, \langle 4,a,5\rangle, \langle 5,a,5\rangle, \langle 5,b,6\rangle, \langle 6,a,5\rangle, \langle 7,a,8\rangle, \langle 8,a,8\rangle, \langle 8,b,7\rangle\}$$

Show or refute that $1 \sim 4 \sim 6 \sim 7$.

Exercise I.3

Show that

- bisimilarity is an equivalence relation
- bisimilarity is is closed to union
- bisimilarity is is closed to intersection

Exercise I.4

A relation R over states of a transition space is a *word bisimulation* if, whenever $\langle p, q\rangle \in R$ e $s \in \mathcal{N}^*$,

$$p \xrightarrow{s} p' \;\Rightarrow\; \langle \exists\, q' \;:\; q' \in S_2 :\; q \xrightarrow{s} q' \;\wedge\; \langle p', q'\rangle \in R\rangle$$
$$q \xrightarrow{s} q' \;\Rightarrow\; \langle \exists\, p' \;:\; p' \in S_1 :\; p \xrightarrow{s} p' \;\wedge\; \langle p', q'\rangle \in R\rangle$$

1. Define formally relation $\xrightarrow{s}$, for any $s \in \mathcal{N}^*$
2. Two states p and q are *word bisimilar* iff there exist a word bisimulation R such that $\langle p, q\rangle \in R$. Discuss whether two states p and q are word bisimilar iff $p \sim q$.

Exercise I.5

In a transition space $\langle S, \mathcal{N}, \downarrow, \longrightarrow \rangle$, the set of *traces* of a state s is the least set including

$$\begin{aligned} \epsilon &\in \mathsf{Tr}(s) \\ \checkmark &\in \mathsf{Tr}(s) \quad \text{if} \quad \downarrow s \\ a\sigma &\in \mathsf{Tr}(s) \quad \text{if} \quad \exists_t \cdot s \xrightarrow{a} t \wedge \sigma \in \mathsf{Tr}(t) \end{aligned}$$

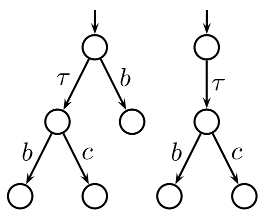
1. Show that two bisimilar states have the same traces? Is the converse true?

Exercise I.6

In what sense can the definitions of *morphism* of transition spaces and *bisimulation* be related?

Exercise I.7

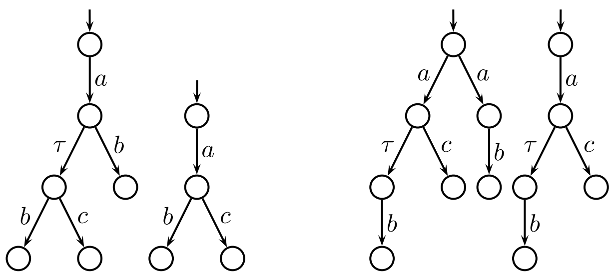
Consider the following transition systems:



Are they branching or rooted branching bisimilar?

Exercise I.8

Which of the following pairs of transition systems are branching and/or rooted branching bisimilar.



Exercise I.9

With regard to the two previous exercises, which τ-transitions are inert wrt branching bisimulation (*i.e.*, for which τ-transitions $p \xrightarrow{\tau} p'$ are $p \approx p'$)?