Leis do Cálculo Relacional

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Axioms: $\cdot$, $\subseteq$

(associativity)	$(R \cdot S) \cdot T \;=\; R \cdot (S \cdot T)$
(identity)	$R \;=\; R \cdot id \;=\; id \cdot R$
($\subseteq$ reflexivity)	$R \subseteq R$
($\subseteq$ transitivity)	$R \subseteq S \wedge S \subseteq T \;\Rightarrow\; R \subseteq T$
($\subseteq$ anti-symmetry)	$R \subseteq S \wedge S \subseteq R \;\Rightarrow\; R = S$
(monotonicity)	$S \subseteq T \wedge R \subseteq U \;\Rightarrow\; S \cdot R \subseteq T \cdot U$

Axioms: $=$

(ping-pong)	$R = S \;\equiv\; R \subseteq S \wedge S \subseteq R$
(indirection)	$R = S \;\equiv\; \forall X \cdot (X \subseteq R \wedge X \subseteq S)$

Axioms: $\cap$, $\circ$

($\cap$ universal)	$X \subseteq (R \cap S) \;\equiv\; X \subseteq R \wedge X \subseteq S$
($\circ$ involution)	$R^{\circ\circ} \;=\; R$
($\circ$ monotonicity)	$R \subseteq S \;\equiv\; R^\circ \subseteq S^\circ$
($\circ$ contravariance)	$(R \cdot S)^\circ \;=\; S^\circ \cdot R^\circ$
(left modular)	$(R \cdot S) \cap T \;\subseteq\; R \cdot (S \cap (R^\circ \cdot T))$

Dedekind Variations

(left modular)	$(R \cdot S) \cap T \;\subseteq\; R \cdot (S \cap (R^\circ \cdot T))$
(right modular)	$(R \cdot S) \cap T \;\subseteq\; (R \cap (T \cdot S^\circ)) \cdot S$
(weak distr)	$(R \cdot S) \cap T \;\subseteq\; (R \cap (T \cdot S^\circ)) \cdot (S \cap (R^\circ \cdot T))$

$\cap$ Laws

($\cap$ associativity)	$(R \cap S) \cap T \;=\; R \cap (S \cap T)$
($\cap$ commutativity)	$R \cap S \;=\; S \cap R$
($\cap$ idempotence)	$R \cap R \;=\; R$
($\cap$ abbreviation)	$R \subseteq S \;\equiv\; R = R \cap S$
($\cap$ cancellation)	$R \cap S \subseteq R,\; R \cap S \subseteq S$
($\cap$ left fusion)	$T \cdot (R \cap S) \;\subseteq\; T \cdot R \cap T \cdot S$
($\cap$ right fusion)	$(R \cap S) \cdot T \;\subseteq\; R \cdot T \cap S \cdot T$

◦ Laws

(reduction)	$R \subseteq R^\circ \;\equiv\; R = R^\circ$
(wrap)	$R \;\subseteq\; R \cdot R^\circ \cdot R$
(dist over $\cap$)	$(S \cap R)^\circ \;=\; R^\circ \cap S^\circ$

Misc

(Dedekind: $T = id$, $R = R^\circ$)	$(R^\circ \cdot S) \cap id \;\subseteq\; (R \cap S)^\circ \cdot (R \cap S)$
(Dedekind: $S = id$)	$R \cap T \;\subseteq\; R \cdot (id \cap (R^\circ \cdot T))$

Kernel laws

(definition)	$\mathsf{ker}\, R \;=\; R^\circ \cdot R$
(duality)	$\mathsf{ker}\, R^\circ \;=\; \mathsf{img}\, R$
(monotonicity)	$R \subseteq S \;\Rightarrow\; \mathsf{ker}\, R \subseteq \mathsf{ker}\, S$
(symmetry)	$(\mathsf{ker}\, R)^\circ \;=\; \mathsf{ker}\, R$
(intro)	$R \;\subseteq\; R \cdot \mathsf{ker}\, R$
($\cdot$ kernel)	$\mathsf{ker}\,(R \cdot S) \;=\; S^\circ \cdot \mathsf{ker}\, R \cdot S$
($\cap$ kernel)	$R^\circ \cdot S \cap id \;\subseteq\; \mathsf{ker}\,(R \cap S)$

Image Laws

(definition)	$\mathsf{img}\, R \;=\; R \cdot R^\circ$
(duality)	$\mathsf{img}\, R^\circ \;=\; \mathsf{ker}\, R$
(monotonicity)	$R \subseteq S \;\Rightarrow\; \mathsf{img}\, R \subseteq \mathsf{img}\, S$
(symmetry)	$(\mathsf{img}\, R)^\circ \;=\; \mathsf{img}\, R$
(intro)	$R \;=\; \mathsf{img}\, R \cdot R$

Order Taxonomy

(reflexive)	$id_A \;\subseteq\; R$
(coreflexive)	$R \;\subseteq\; id_A$
(transitive)	$R \cdot R \;\subseteq\; R$
(symmetric)	$R \;\subseteq\; R^\circ$
(anti-symmetric)	$R \cap R^\circ \;\subseteq\; id_A$
(connected)	$R \cup R^\circ \;=\; \top_A$
(entire)	$id \;\subseteq\; \mathsf{ker}\, R$ (total relation)
(simple)	$\mathsf{img}\, R \;\subseteq\; id$ (partial function)
(surjection)	R° entire
(injection)	R° simple

Coreflexives

(symm & transitive)	$R \;=\; R^\circ \;=\; R \cdot R \;=\; R \cap id$
(cancellation)	R coreflexive $\;\Rightarrow\; (R \cdot T \subseteq T) \wedge (T \cdot R \subseteq T)$
(distributivity)	R coreflexives $\;\Rightarrow\; (R \cdot T) \cap S = R \cdot (T \cap S)$

Relational Operators as Galois Conections			
$(f\ X) \subseteq Y \equiv X \subseteq (g\ Y)$			
Description	$f = g^\flat$	$g = f^\sharp$	**Obs.**
converse	$(\_)^\circ$	$(\_)^\circ$	
right-division	$(\cdot R)$	$(\ \ / R)$	$\ldots$ over R
left-division	$(R \cdot)$	$(R \setminus\ \ )$	R under $\ldots$
shunting rule	$(f \cdot)$	$(f^\circ \cdot)$	NB: f is a function
"converse" shunting rule	$(\cdot f^\circ)$	$(\cdot f)$	NB: f is a function
range	rng	$(\cdot \top)$	lower $\subseteq$ restricted to coreflexives
domain	dom	$(\top \cdot)$	lower $\subseteq$ restricted to coreflexives
implication	$(R \cap\ \ )$	$(R \implies\ \ )$	Note that $(R \implies\ ) = (\neg R \cup\ \ )$
difference	$(\_ - R)$	$(R \cup\ \ )$	
definition	$f\ X = \bigcap\{Y \mid X \subseteq gY\}$	$g\ Y = \bigcup\{X \mid f\ X \subseteq Y\}$	
distributive property	$f(X \cup Y) = (f\ X) \cup (f\ Y)$	$g(X \cap Y) = (g\ X) \cap (g\ Y)$	$f(\bigcup_i X_i) = \bigcup_i (f\ X_i)$ $g(\bigcap_i X_i) = \bigcap_i (g\ X_i)$