

Formal Methods in Software Engineering

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Software Engineering

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 - ... the larger the project, the worse the picture ...



Some Development Catastrophes

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CONFIRM travel information system	\$ 160 million
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[Sci. American, Sep. 1994]

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25% of all large systems are cancelled



[Sci. American, Sep. 1994]

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- deficient management determined to push through



Critical Software Failures

US Telephone Network

[switching software failures, 1991]

Bank of New York

[90 minutes failure, Nov. 20 1985]

lost information on \$ 32 b in transactions

forced to borrow \$ 23.6 b from the FR

at a cost in interest of \$ 5 m]

GM Detroit Automated Factory

[1 year working at half its capacity]


THERAC-25 Radiotherapy

[2 killed by overdose]

[*Digital Woes*, Wiener, L., 1993]



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- But besides costs ... software industry has to deal **mainly** with **safety-critical** and **mission-critical** applications.







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But is Software Engineering (SE) ... Engineering?

Hardware vs Software Engineering

Reusable Components	“Flat” & Unrelated Tons of Code
Catalogue of specifications	... formal?
Refinement Methods	Why reuse, if it is easy to begin again?
Development stations	Debug = (Edit;Compile;Test)*
Standard Methodologies and Notation	Proliferation ...
Production and Maintenance: <ul style="list-style-type: none"> Production Plans & Data Bases Components' Families — classification, equivalences, etc.	Cubic meters of <ul style="list-style-type: none"> unreadable programs unhelpful manuals



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CASE tools are mostly oriented toward the *production* process (e.g., goal analysis, planning, version control, etc.), but have modest success in addressing the *product* itself.



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There is a need for a software technology with a sound mathematical basis, coping with composition and refinement, in which a program would be unacceptable unless accompanied by a guarantee that it respects its specified behaviour.



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systems' descriptions



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Precise, abstract descriptions \equiv **mathematical models**



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From school physics, recall a basic **problem solving strategy**:



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



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From school physics, recall a basic **problem solving strategy**:

-  Understand the problem
-  Create a **mathematical model**
-  Reason within the model
-  Calculate a **solution** (\equiv an **implementation**)



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Suitable **models** are:





abstract (\equiv *concise and precise descriptions*)



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
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\rightsquigarrow **Formal Specification & Development Methods**



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↪ **Formal Specification & Development Methods**

- *Model-Oriented*: VDM, Z, B, RAISE, CAMILA, ...
- *Property-Oriented*: CLEAR, OBJ, ...



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Links:

www.fmeurope.org (includes an applications database)

www.comlab.ox.ac.uk/archive/formal-methods (huge FM Archive)

shemesh.larc.nasa.gov/fm.html (NASA FM Page; generalistic)



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
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*‘There is a big difference between good, sound reasons, ...
... and reasons that sound good’ (Haldane)*



Why Maths?



Why Maths?

and God said,

$$\begin{aligned}
 E &= hf = hc/\lambda, \quad \Delta V_0 = hf - W, \quad E = mc^2, \quad E = P^2/2m, \quad \Psi(x,t) = \int_{-\infty}^{\infty} A(k) e^{i(kx - \omega t)} dk \\
 E &= hc/\lambda, \quad \Psi(x,t) = e^{i(kx - \omega t)} \int_{-\infty}^{\infty} A(k) e^{i(kx - \omega t)} dk, \quad \lambda = \frac{h}{mv}, \quad E = p^2/2m \\
 \Psi(x,t) &= e^{i(kx - \omega t)} \int_{-\infty}^{\infty} A(k) e^{i(kx - \omega t)} dk, \quad \lambda = \frac{h}{mv}, \quad E = p^2/2m \\
 E &= \hbar^2 k^2 / 2m, \quad E = \hbar \omega = \hbar^2 k^2 / 2m, \quad m_{eff} = \frac{m}{\sqrt{1 - v^2/c^2}}, \quad \frac{\hbar^2 k^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} = \hbar^2 \frac{\partial \Psi}{\partial t} \\
 \frac{\partial^2 \Psi}{\partial x^2} + \frac{2mi(E - V)}{\hbar^2} \Psi &= 0, \quad k^2 = \frac{2mi(E - V)}{\hbar^2}, \quad \lambda = \frac{h}{\sqrt{2m(E - V)}}, \quad E = \frac{1}{2} \hbar^2 k^2 \\
 E &= \frac{\hbar}{2m} \left(\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} \right) = \frac{\partial \rho}{\partial t} \Psi, \quad \rho = \Psi^* \Psi, \quad \frac{d^2 x}{dt^2} + \frac{\hbar}{m} \frac{d^2 \Psi}{dt^2} = 0 \\
 J &= \frac{1}{r \sin \theta} \left[\frac{\partial H}{\partial \theta} \sin \theta - \frac{\partial H}{\partial \theta} \right] \frac{\partial f}{\partial \theta} \frac{\partial f}{\partial \theta} + \frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial f}{\partial \theta} \frac{\partial H}{\partial \theta} \right] \frac{\partial f}{\partial \theta} + \frac{1}{r} \left[\frac{\partial f}{\partial \theta} \frac{\partial H}{\partial \theta} \right] \frac{\partial f}{\partial \theta} \\
 &= \frac{\hbar^2}{2m} \left(\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} \right) + V \Psi = E \Psi, \quad V = -\frac{G}{4\pi a_0 r} - \frac{G}{4\pi a_0 r} \frac{1}{\sqrt{x^2 + y^2 + z^2}} \\
 \nabla^2 f &= \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \phi^2}, \quad f = \lim_{\Delta Q \rightarrow 0} \frac{q^* \Delta Q}{\Delta Q} \\
 \nabla \cdot D &= \frac{1}{h_1 h_2} \left[\frac{\partial}{\partial x} (h_2 h_3 D_x) + \frac{\partial}{\partial y} (h_1 h_3 D_y) + \frac{\partial}{\partial z} (h_1 h_2 D_z) \right] \\
 \mu_r &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{r} \frac{\partial V}{\partial r} \left(\frac{1}{r} \frac{\partial V}{\partial r} \right) dr^2 \sin^2 \theta d\theta d\phi = \frac{4\pi r^2}{\ln \left(\frac{r_2}{r_1} \right)} \left(r - \frac{\ln 2}{2} \right) \sin^2 \theta \\
 J_n(t) &= \sum_{k=0}^{\infty} \frac{(-1)^k 2^{k+1} t^{k+1}}{k! \Gamma((m-1)+1) 2^{k+1} t^{k+1}}, \quad J_{-m}(t) = \sum_{k=0}^{\infty} \frac{(-1)^k 2^{k+1} t^{k+1}}{k! \Gamma((m-1)+1) 2^{k+1} t^{k+1}} \\
 \oint_C \vec{D} \cdot d\vec{l} &= emf = - \int_C \frac{\partial \vec{B}}{\partial t} \cdot d\vec{l}, \quad \oint_C \vec{E} \cdot d\vec{l} = - \int_C \left(\vec{E} + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{l}, \quad \oint_C \vec{D} \cdot d\vec{l} = Q = \int_V \rho \cdot dV \\
 E_r &= \frac{1}{4\pi\epsilon_0} \left(\frac{\mu}{r^2} + \frac{1}{\mu\omega^2 r^2} \right) \cos \theta, \quad E_\theta = \frac{1}{4\pi\epsilon_0} \left(\frac{\mu}{r} + \sqrt{\frac{\mu}{r}} \frac{1}{r^2} + \frac{1}{\mu\omega^2 r^2} \right) \sin \theta \\
 F(r, \theta, t) &= \frac{-\alpha \mu^2}{4\pi\epsilon_0} \sin \theta \sin^2 \theta \cos \theta - \mu^2 \sqrt{\mu r} \sin \theta, \quad H(r, \theta, t) = \sqrt{\frac{\epsilon}{\mu}} E_r \sin \theta = \sqrt{\mu \omega} \mu^2 \dots
 \end{aligned}$$

and there was light.



Need for Tool Support



Most systems grow up from an unstructured collection of informal requirements



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There is a need for tools to validate software formal descriptions:
*type checkers, syntax-oriented editors, theorem provers, proto-
typing environments...*

