Universidade do Minho
Escola de Engenharia
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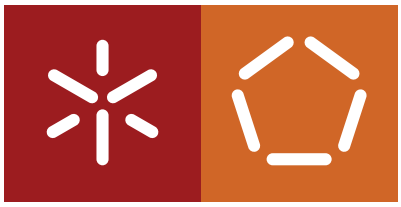
Yoan David Ribeiro

Validation of IEC 61131-3 Programmable Logical Controllers in KeYmaera

October 2015



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Validation of IEC 61131-3 Programmable Logical Controllers in KeYmaera

Master dissertation
Master Degree in Computing Engineering

Dissertation supervised by
Luís Soares Barbosa
Alexandre Madeira

October 2015

ACKNOWLEDGEMENTS

This page is always a little awkward and some might think embarrassing but let me tell you:

There is no shame in being grateful for someone or something, on the contrary, gratitude is what makes us happy, it is what make us bond with one another, it enriches our lives. I could go on with the benefits of gratitude but I would not be thanking anyone that way.

So, this dissertation would not be possible without the careful watch of my supervisors Prof. Luís Barbosa and Alexandre Madeira. I have learnt greatly with them and I am grateful for the opportunity that they gave me, their patience and positive input, many thanks. I have to thank Prof. José Nuno Oliveira for the inspiration that he continuously gives me and so many other students, to do our best and pursue our passions. I am also grateful to Prof. António Martins and Renato for their opinions, positive feedback and being always ready to help.

Of course I am grateful for my friends and their will to help me whenever they can, specially to the MFES Way crew, with no particular order: Ana, Damien, Paulo, Lobo, Luís, Vitor, Fábio Fernandes, Catarina, Fábio Esteves, João, Telma, Nuno, Miguel, Rogério thank you so much for helping me and becoming a better version of myself!

And last but definitely not least, my family. Without them I would not be where I am; they are the foundation of the person I am today. Dad, Nela, Sara, Diego, Patrícia, Ruben and so on. I am lucky to call you my Family, thank you for everything, I am forever grateful for what you have done for me.

Thank you very much to everyone that in one way or another helped me in this journey so far and do not forget there is no shame in being grateful because, as Cicero said:

"Gratitude is not only the greatest virtue, but the parent of all the others"

ACKNOWLEDGEMENTS

This work is financed by the ERDF - European Regional Development Fund through the COMPETE Programme (operational programme for competitiveness) and by National Funds through the FCT - Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) within project FCOMP-01-0124- FEDER-028923

ABSTRACT

PLCs (Programmable Logical Controllers) are embedded computers built specifically for the industrial environment, and used for the automation of industrial processes. These systems are typically developed resorting to programming languages defined in the IEC 61131-3 standard, which includes two textual and three graphical programming languages. IEC 61131-3 has become widely established in recent years as the programming standard for industrial automation. Actually, a wide range of small to large PLC manufacturers offer programming systems that are based on it. Methods to formally analyse PLCs developed in this framework are needed, namely to provide timing guarantees as well as to quantify the dependability parameters of the resulting applications. In this dissertation we propose to tackle these issues resorting to the KEYMAERA verification tool and its associated dynamic logic for hybrid systems. KEYMAERA is a verification tool for hybrid systems supporting differential dynamic logic (dL), which combines deductive, real algebraic, and computer algebraic prover technologies. dL is a suitable logic to deal with hybrid systems, that combines first-order dynamic logic, whose atomic programs are variables assignments (to represent discrete evolutions), with ordinary differential equations (to represent continuous evolutions). The dissertation is devoted to the design and development of a practical validation framework for IEC 61131-3 PLCs based on KEYMAERA. This involves the development of a translator from the controller program to the discrete (cyber) component of a KEYMAERA model and the specification of contextual conditions (e.g. measured by sensors in real scenarios) as the continuous (physical) component of the model. The whole research guided guided by some case studies.

RESUMO

PLC (Programmable Logical Controllers) são computadores embebidos construídos para o ambiente industrial, e usados em automação de processos industriais. Estes sistemas são tipicamente programados recorrendo às linguagens de programação definidas na norma IEC 61131-3, que inclui duas linguagens de programação textuais e três gráficas. Actualmente, uma grande variedade de pequenos e grandes construtores de PLC oferecem sistemas de programação baseados nessas linguagens. Métodos para uma análise formal de PLCs construídos nessa framework são necessários, nomeadamente para providenciar garantias de tempo assim como a quantificação de parâmetros de confiança das aplicações resultantes. Nesta dissertação abordam-se estes problemas recorrendo à ferramenta de verificação KEYMAERA e à sua lógica dinâmica associada para os sistemas híbridos. KEYMAERA é uma ferramenta de verificação para sistemas híbridos que suporta lógica dinâmica diferencial (dL), combinando tecnologias *prover deductive, real algebraic e computer algebraic*. dL é uma lógica dinâmica de primeira ordem para programas híbridos. dL é uma lógica adequada ao tratamento de sistemas híbridos, que combina lógica dinâmica de primeira ordem, cujos programas atómicos consistem em valorações de variáveis (para modelar evoluções discretas) com equações diferenciais ordinárias (para modelar evoluções contínuas). Esta dissertação foca-se na concepção de um framework de validação prática para PLCs IEC 61131-3 baseada em KEYMAERA. Tal envolve a construção de um tradutor para o programa do controlador para um componente discreto (cíber) de um modelo KEYMAERA e a especificação de condições de contexto (e.g. em casos reais, medidas por sensores) como a componente contínua (física) do modelo. A investigação foi guiada por alguns casos de estudo.

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INTRODUCTION

The world, nowadays, relies on software for everything for instance in cars, aircrafts, medical tools and other domains of application. From this extensive use of software a question arises: Can we trust the software running on those systems? A honest answer to this question is: it depends. Some will be safe, and others will not and that is because not every entity responsible for building software uses formal methods. Formal methods are more crucial than ever because we, human beings need to be sure that the software that we use in our every day life is reliable and safe to use. This is only possible if mathematical based techniques are used to specify and verify software. This dissertation aims to apply formal methods to validate a subset of the *IEC 61131-3* standard, for programmable logic controllers, programming languages using KEYMAERA which is a theorem prover suitable for some classes of *cyber-physical systems*.

1.1 MOTIVATION

Programmable Logic Controllers (PLCs) are embedded computers used for industrial automation and programmed according to the standard IEC 61131-3 (John and Tiegelkamp, 2010). As any other system, PLCs have to, somehow, be verified and tested.

Simulation focuses mainly in the behavioural part of a program/system and usually provides a visual representation of the system which makes it more tangible. Consequently, this technique constitutes the usual method used to verify if the PLC program follows its behaviour. Note however this technique cannot prove, for instance, that a given behaviour will always or never occur, leading to a incomplete verification.

Besides simulation, model-checking is also widely used technique in formal methods. It focuses on proving that a set of proprieties meets the specification given by a model. Although a powerful method to find errors in a model, model-checking only proves correctness to a finite-state system which implies that it is often not sufficient to prove that the model is without fault.

The technique used in this dissertation is verification through a theorem prover. When given a model, which represents the system or more specifically the PLC program, this method goes through all the possibilities to verify if the given model follows the set of proprieties.

Furthermore, it is well known that PLCs interact with the world through sensors and evaluate the information provided by the sensor to discretely change their behaviour if necessary. Therefore, these systems can be classified as *cyber-physical systems*.

Cyber-physical systems are systems which contain computational elements that interact and control physical entities. These systems are also known as embedded systems with a emphasis in the intense link between the computational and physical elements. Consequently, they exhibit both continuous dynamics and a discrete dynamics. To prove specific properties about such systems, the theorem prover KEYMAERA uses differential dynamic logic ($d\mathcal{L}$) (Platzer, 2010), and combines a deductive prover and computer algebraic solvers technologies.

After recognizing that programmable logical controllers can be seen as cyber-physical system, the next step is to set up a framework for their validation. The step will consist of transforming the programs which are ran by PLCs to KEYMAERA models and resort to that tool to prove them.

Summing up, this dissertation aims at showing how to verify PLC programmed according to the standard IEC 61131-3 using KEYMAERA.

1.2 CONTEXT

The Nasoni project, which gives the context to this dissertation, proposes to study software composition in the domain of the coordination paradigm which integrate different and often loosely coupled software entities, as is the case of PLCs.

The focus of this dissertation is to translate the languages used to build programmable logic controllers into KEYMAERA formulas. The strategy to achieve such a goal resorts to a set of case studies to better understand the differences and, more importantly, the similarities between the languages of the IEC-61131-3 and KEYMAERA hybrid programs, and then to design a program able to translate most of the norm to KEYMAERA.

The IEC-61131-3 determines how PLCs are built, and not only the language in which the controllers are programmed, but also, how the PLC programs should be designed. The norm defines five different languages to build PLC programs:

- Instruction List;
- Structured Text;
- Function Block Diagram;
- Ladder Diagram;
- Sequential Function Charts.

The first two are textual and the remaining three are graphical (although sequential function charts has also a textual representation).

After understanding how PLC programs are built within the standard, it is necessary to design their translation to a KEYMAERA hybrid program. Certainly, the translation process has various steps, depending on the target language. In particular, this dissertation will focus mainly on the graphical language *sequential function charts* because of its resemblance to automata, which eases the translation process.

1.3 CONTRIBUTION

The main contribution of this dissertation is to provide a basic tool to verify PLC programs by generating a KEYMAERA model based on the program. The method underlying such transformation is the formalisation of the sequential function chart with conditional ordinary differential equations, *CODE* for shorts, discussed in (Nellen and Ábrahám, 2012).

To achieve such a goal, an annotation language was designed to enrich the SFC language of Beremiz, which is the IDE in which PLC programs are developed.

With the tool, two case studies were designed to test and evaluate its potential.

1.4 ROADMAP

After exposing the motivation, context and contribution of this dissertation, the remaining chapters are arranged as follows, (see Figure 1):

CHAPTER 2 covers the background for this dissertation, namely, the languages for the PLC programs, cyber-physical systems and respective models as well as the KEYMAERA theorem prover. Furthermore, it provides a solid overview of the tools used to build PLC programs emphasising the Beremiz IDE since it was the one used throughout this dissertation. This includes the methods used to model cyber-physical systems and the various formalisations of the PLC programming languages.

CHAPTER 3 is the core chapter of this dissertation. It details the overall approach and tool to translate the SFC language annotated to KEYMAERA.

CHAPTER 4 AND 5 are a support chapter, describing the case studies of the water heating tank and pH plant and how to go from the SFC code with the annotations to KEYMAERA hybrid program. The latter is used to prove a number of safety proprieties.

CHAPTER 6 discusses the conclusions of this dissertation.

APPENDIX A is an auxiliary work where it is show case the translation from structured text to KEYMAERA.

APPENDICES B, C AND D contain the Haskell data structure of the different stages of the translation to build insight on how the tool was design. Appendix E and F contain the XML file of the case studies obtained by the Beremiz IDE.

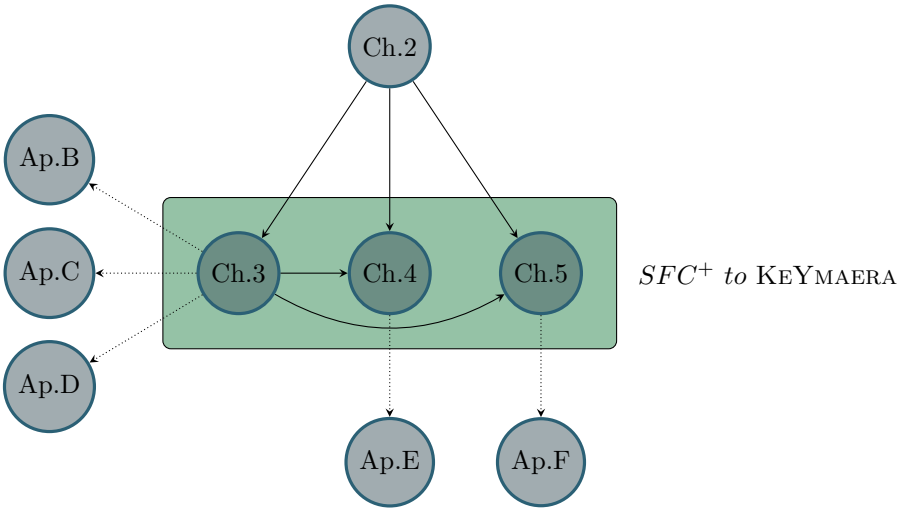


Figure 1.: Roadmap of the dissertation.

2

BACKGROUND AND STATE OF THE ART

This chapter reviews the state of the art and background which serves as a warm up for the remaining of the dissertation. It covers the basic concepts necessary to a full understanding of this document. As well as the different tools and ideas which have been developed and implemented in the areas addressed this dissertation. These are the *cyber-physical systems* (hybrid systems), *automation*, more particularly, *programmable logic controllers* and formalisations of standard IEC 61131-3.

2.1 PROGRAMMABLE LOGIC CONTROLLER

Industrial automation involves the use of multiple control systems, which are devices that administer, direct, command or adapt the behaviour of other devices or systems. For operating equipment, namely manufacturing processes, industrial machinery and many others often requires none or minimum human intervention, the most used control systems are the programmable logic controllers.

Programmable logic controllers (John and Tiegelkamp, 2010) are digital computer devices designed for multiple analogue and digital inputs and output arrangements. They must be resistant to vibrations and impacts, durable to extended temperature ranges and immune to electrical noise. The programs which are responsible for the machine usually are stored in battery-backed-up or ROM. PLCs are an example of hard real-time systems due to the limited time in which the output must be produced in response to input conditions.

IEC 61131-3 (John and Tiegelkamp, 2010) is the standard used to describe the PLC programming languages. The standard provides comprehensive concepts and guidelines for creating PLC projects.

2.1.1 *IEC 61131-3*

Program Organisation Unit (POU)

This component may be seen as a block which runs a set of instructions. It is the smallest independent software unit. There are three types of POU:

- Function (FUN), units of this type behave as functions, always returning the same result, for the same input parameters, *i.e.* they have no memory;

- Function block (FB), units of this kind have their own data record which saves status information that the programmer can instantiate;
- Program (PROG), the "top" POU of a PLC program, it has the ability to access the PLC I/Os and make them accessible to other POU.

Each of these program organisation units is divided in two main parts: The *declaration part* which, as the name suggests, is where the variables are declared and, the *code part*, where the PLC program is coded. The variables can be *global*, *input*, *output*, *input/output* and others and have to inhabit a specific data type: *Byte*, *Bool*, *Integer* etc.

The IEC 61131-3 provides three textual languages and three graphical languages for writing application programs (the code part of a POU). The textual languages are (John and Tiegelkamp, 2010):

- Instruction List (IL);
- Structured Text (ST);
- Sequential Functional Chart (textual version of SFC).

And the graphical languages are:

- Ladder Diagram (LD);
- Function Block Diagram (FBD);
- Sequential Function Chart (graphical version of SFC).

The *instruction list* is an assembly like language, which gives to the programmers the opportunity to work at a relatively low level, meaning that there is nearly no abstraction from the PLC instruction set architecture. IL is largely used and is frequently employed as an intermediate language to which other textual and graphical languages are translated.

The second language, structured text, consists of a sequence of statements, just as a classic imperative languages, namely, PASCAL and C. These statements consist of a combination of keywords which control the program execution and expressions which are operators/functions and operands to be evaluated at run time .

The Table 1 sums up the overall statements of the ST language.

Keyword	Description	Example	Explanation
:=	Assignment	d := 10;	Assignment of a value to an identifier.
	Call of an FB	`FBName(Par1 := 10,` `Par2 => Res);`	Call of other POU of type FB including its input parameters ":=" and its output parameters "=>"
RETURN	Return	`RETURN;`	Leaves the current POU and return to the calling POU

IF	Selection	`IF d < e THEN f := 1;`
		`ELSIF d = e THEN f := 2;`
		`END_IF`
			Selection of alternatives by means of Boolean expressions.

CASE	Multi-selection	`CASE f OF`
		`1: g := 11;`
		`2: g := 12;`
		`ELSE g := 13;`
		`END_CASE;`
			Selection of a statement block depending of the value of the expression "f".

FOR	Iteration (1)	`FOR h:=1 TO 10 BY 2 DO`
		`f[h/2] := h;`
		`END_FOR;`
			Multiple loop of a statement block with a start and an end condition and an increment

WHILE	Iteration (2)	`WHILE m > 1 DO`
		`n := n /2;`
		`END_WHILE;`
			Multiple loop of a statement block with an end condition at the beginning

REPEAT	Iteration (3)	`REPEAT`
		`i := i*j;`
		`UNTIL i < 1000`
		`END_REPEAT;`
			Multiple loop of a statement block with an end condition at the end

EXIT	End of loop	`EXIT;`	Premature termination of an iteration statement
;	Dummy statement		

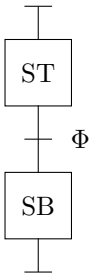
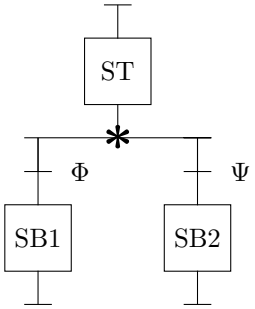
Table 1.: ST code and explanation.

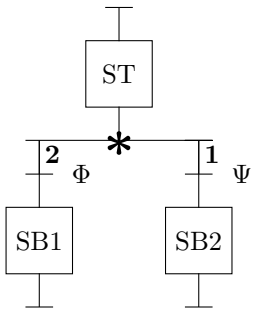
The *functional block diagram* and the *ladder diagram* languages come originally from the area of signal processing, where integer and/or floating point values are crucial.

FBD language is graphical language in which, as the name suggests, the main components are function blocks. These represent at the left side the input gates, which are the input variables, and output gates at the right side. Two function blocks can communicate if the output of the first is connected to the input of the second one. This language is popular since its appearance is similar to that of logic circuits.

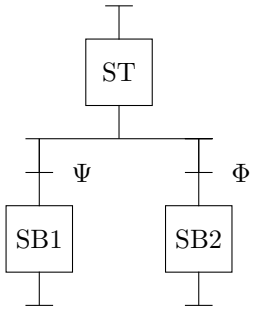
The LD language is also a graphical language which was primarily designed for processing boolean signals. The language was initially called ladder logic because the method used to document the design and construction of relay racks, in manufacturing and process control. This logic evolved into the programming language known today and is part of the IEC.

Finally, *Sequencial Function Chart* is a language defined to break down a complex program into smaller and more manageable units, as well as to describe the control flow between those units. With SFC, it is possible to design sequential and parallel processes (see Table 2). This graphical language is noticeably similar to that of state machines.

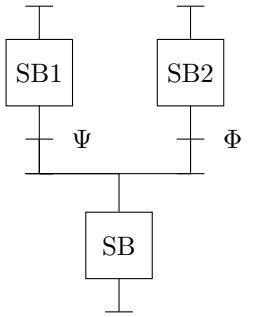
Graphical object	Name and explanation
	**Single Sequence:** Alternation of step → transition → in series. ST is deactivated and SB becomes active, as soon as Φ evaluates to TRUE
	**Divergent Path:** Selection of exactly one sequence, evaluated from left to right. The first one which evaluates to TRUE from left to right, deactivates ST and activates the corresponding step.



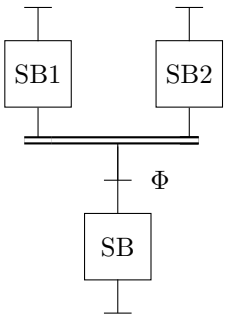
Divergent path with user-defined priority: Selection of exactly one sequence, evaluated by priorities defined by the user. The first to evaluate to TRUE, with the priority order defined the user, deactivates ST and activates the corresponding step.



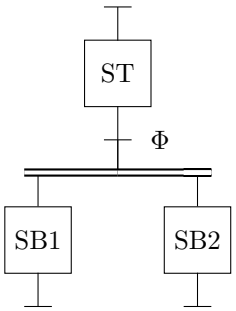
Divergent path under user control: Selection of exactly one sequence. Transition (Φ and Ψ) must be mutually exclusive and the first to evaluate to TRUE in those conditions, deactivates ST and activates the corresponding step.



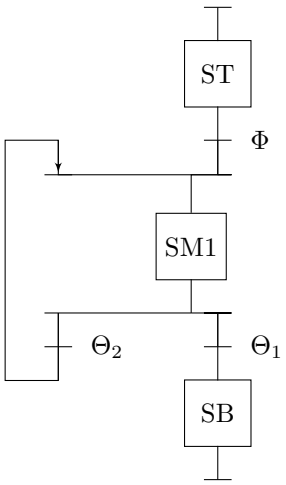
Convergence of sequence: The divergent paths are combined. When one of the STn is active and the corresponding successor transition becomes TRUE, STn deactivates and SB is initiated.



Convergence of simultaneous sequences: The path of simultaneous path are combined. When all STn are active and the transition Φ evaluates to TRUE then all STn are deactivated and SB is activated.



Simultaneous sequences: Simultaneous activation of all connected steps. ST is deactivated when Φ evaluates to TRUE and all subsequent steps connected via Φ become active simultaneously.



Sequence Loop (Branch back) Is a combination of the already described sequences

Table 2.: SFC graphical code and explanation.

2.1.2 *Tools to build programs for PLCs*

In computer programming, it makes no big difference if what is required is a program for a cloud application or a program for controllers. *Integrated Development Environment* (IDE) often comes handy and PLC programs are not too much different. For PLC development, the search of IDEs was focused on two criteria:

1. The IDE respects the IEC 61131-3 standard;

2. The use the IDE does not rely on the existence of a PLC.

There are many IDEs which fulfil the criteria mentioned above, for instance, CODESYS (Controller Development System [1]), and OpenPCS Automation Suite [2]. These IDEs allow programmers to produce PLC code using the five languages of the standard and still includes a sixth one, the CFC (Continuous Function Chart). CODESYS compiles the programs into binary code to be transferred to a PLC. However none of these are Open Source. The one IDE which still satisfy the above criteria and is Open Source, is Beremiz [3]. Therefore, it was the chosen platform to develop PLC programs in this dissertation.

2.1.3 *Beremiz*

The Open Source IDE Beremiz is a multi-platform program that allows the writing of automation programs always in accordance with the IEC 61131-3 standard. The public availability of the software is granted under a GNU GPLv2.1 license. The software has the following built in modules:

- PLCOpen Editor - IEC 61131-3 Program editor;
- MATIEC - Compiler of IEC 61131-3 to ANSI-C;
- CanFestival - CANOpen Framework for interface with physical I/O;
- SVGUI - Integration tool for Human-Machine Interfaces (HMIs).

The MATIEC compiler converters programs which are built using the standard languages to equivalent C code, and follows four usual steps: lexical analyser, syntax parser, semantics analyser and code generator (Tisserant et al., 2007).

1 CODESYS software: Available at: www.3s-software.com
2 Open OCS Automation Suite: Available at: www.systec-electronic.com/en/products/automation-components/development-and-configuration-tools/openpcs-iec-61131-3-automation-suite
3 Beremiz website: www.beremiz.org

2.2 CYBER-PHYSICAL SYSTEMS

Cyber-physical systems features both discrete and continuous changes. Typically, the expression refers to systems or embedded controllers for physical systems, for instance robots, aircrafts, trains, cars, and others, whose continuous dynamics are governed by physical principles such as fluid/thermo dynamics or classical mechanics. These often are critical systems, since if a malfunctioning of the system occurs, it can endanger lives.

2.2.1 *Hybrid Automata*

The most popular formalism behind a cyber-physical, or hybrid system is the *hybrid automata* (HA). These automata exhibit on the nodes continuous behaviour, represented by differential equations, which can be restricted by specific invariants. The edges, which may contain attributions and jump conditions to other nodes. These additional features reflect the behaviour of the environment in each node. Hybrid automata can be seen as a generalisation of timed automata in (Alur and Dill, 1994). Its formal definition was proposed in (Henzinger, 1996) and the following definition is an adaptation presented in (Nellen and Ábrahám, 2012) based in paper (Alur et al., 1995)

Definition 1 (Hybrid automata)**.** *A* hybrid automaton

$$H = (Loc, Var, Edge, Act, Inv, Init)$$

consists of a tuple where

- Loc *is a finite set of locations or control modes;*
- Var *is a finite set of real-valued variables and V a set of valuations V. A valuation* $v \in V$, $v : Var \to \mathbb{R}$ *assigns a value to each variable. A state* $s \in Loc \times V$ *pair a location and a valuation. The set of all states is denoted by* Σ*;*
- $Edge \in Loc \times 2^{V^2} \times Loc$ *is a set of edges;*
- Act *is a function assigning a set of time-invariant activities* $f : \mathbb{R}_{\geq 0} \to V$ *to each location, which implies,* $\forall_{l \in Loc} : f \in Act(l) \to (f + t) \in Act(l)$ *where* $(f + t)(t') = f(t + t')$ *forall* $t, t' \in \mathbb{R}_{\geq 0}$*. These represents the* $flow$ *function in a way such, that given a location it returns the set of differential equations of the location;*
- $Inv : Loc \to 2^V$ *is a function assigning an invariant to each location;*
- $Init \subset \Sigma \times V$ *is a set of initial states.*

The typical example of an hybrid automata is a thermostat (Figure 2) which turns on a heater to heat the room until the temperature is below nineteen degrees and the system turn it off the heater whenever the latter is greater than twenty one degrees.

The variable x represents the temperature of the room which is measured by a sensor. The control mode *Off* represents a scenario in which the heater is off, the temperature falls according to the differential equation (flow condition) $x' = -0.1x$. In control mode *On*, the temperature increases following the flow condition $x' = 5 - 0.1x$. At the beginning, the heater is off and the temperature starts at twenty degrees. The jump condition $x < 19$ represents the possibility of the heater being switched on immediately after the temperature falls bellow nineteen degrees. Note that according to the invariant condition $x \geq 18$, as soon the temperature falls to eighteen degrees the heater has to be turned on.

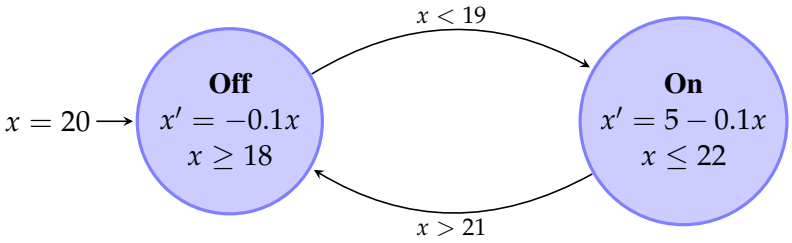


Figure 2.: An hybrid automata representing a thermostat.

2.2.2 *Specification languages for cyber-physical system*

As any other systems, cyber-physical systems can be described using formal specification languages. A typical example of such language is HCSP (Chaochen et al., 1996), an extension of CSP (Hoare, 1978), introduced by Zhou in the mid-nineties. This specification language enriches CSP with continuous variables that evolve according to differential equations. It also handles interruption constructs, activated by communication detections or timeouts. Being an extension to CSP, HCSP inherits its operators, for instance, sequential and parallel composition and interleaving.

The semantics of HCSP is given in a from extended duration calculus (Liu et al., 2010) provides a calculus for a subset of HCSP.

Another specification language for hybrid systems is a extension to EVENT-B language (Abrial, 2010) referred as HYBRID EVENT-B.

The HYBRID EVENT-B language, which is based on a language that gives relevance to events and refinements, enriches Event-B with continuous variables, also called *pliant variables*. These *pliant* along with the discrete variables, become time-dependent, which enforces that the values of the variables to be given as functions of time. The continuous behaviour is described through differential equations or time-dependent expressions. HYBRID EVENT-B, as a specification language for hybrid systems, was assessed in a number of case studies (Banach and Butler, 2013), (Banach and Butler, 2014). However, a tool supporting verification of models in HYBRID EVENT-B is yet to be implemented.

Another example of specification language for cyber-physical systems is an extension of the ACTION SYSTEM language which was proposed, originally, by (Back and Kurki-Suonio, 1988), and is based on the guarded command language of (Dijkstra, 1975) and enriched by (Rönkkö et al., 2003) with continuous state transition (differential actions) whose behaviour is described by differential equations. This approach creates conflict with the support system of ACTION SYSTEM for "step-by-step" refinement, because, differential equations have usually unique solutions. This problem is resolved with the introduction of differential predicates, proposed by the authors, hence bringing non-deterministic continuous evolution to the specification language.

The verification of specifications, written in ACTION SYSTEM can be done up to a certain degree, as defined in (Rönkkö and Li, 2001).

The modelling language SHIFT by (Deshpande et al., 1997) focuses on reconfigurable networks of cyber-physical systems, more concretely on systems with a set of hybrid components whose interaction patterns may change over time. Models written in SHIFT can be translated into C language, providing a functional program which simulates the modelled system. It is also worth mentioning that, this language was used to specify and analyse a system of vehicles in the highway system (Deshpande et al., 1997).

And last but not least, KEYMAERA by (Platzer, 2010) is a verification tool for hybrid systems, that blends deductive, real algebraic and computer algebraic prover technologies. It is a semi-automatic and interactive theorem prover. KEYMAERA supports differential dynamic logic ($d\mathcal{L}$). The theorem prover supports hybrid systems with non-linear discrete jumps, non-linear differential equations, differential-algebraic equations, differential inequalities, and systems with non-deterministic behaviour. For automation, KEYMAERA implements a free-variable sequent calculus and automatic proof strategies that decompose, symbolically, the system specification. Such a compositional verification is the key to scale up verification to big systems (by verifying the properties of their subsystems). KEYMAERA was used to verify parametric hybrid systems and has been successful in doing so, more particularly on case studies such as train control ((Platzer and Quesel, 2009) and (Platzer, 2010)), car control ((Loos et al., 2011) and (Loos and Platzer, 2011)) and air traffic ((Platzer and Clarke, 2009), (Platzer, 2010) and (Jeannin et al., 2015))

2.2.3 *Model Checkers for Hybrid Automata*

Model checkers are powerful tools to prove proprieties about a given model by exhaustive (or symbolically controlled) search over a the state space.

This alternative verification technique is also used in cyber-physical systems, examples are HYTECH (Henzinger et al., 1997), PHAVer (Frehse, 2005), UPPAAL-SMC (David et al., 2012). There is several approaches, such as the decomposition of hybrid automata (Lynch et al., 2003), abstraction methods and approximations techniques (Henzinger, 1996), (Henzinger et al., 1994) and (Alur et al., 1995), to

deal with state explosion problem *i.e.* the number of state grows exponentially, as well as complexity of the verification procedure making the technique unusable.

2.2.4 KeYmaera

KeYmaera (Platzer, 2010) is a semi–automatic verification tool for hybrid systems. It uses a deductive verification approach verifying the model by proof and consequently does not require a finite-state abstractions.

The foundation behind the tool is $d\mathcal{L}$, a first–order dynamic logic whose transition semantics subsumes discrete/continuous behaviour. More specifically, the $d\mathcal{L}$ logic is a *dynamic logic* (Harel et al., 2000), *i.e.* a modal logic whose original purpose is to prove computer programs. Thus, $d\mathcal{L}$ incorporates the typical operators of dynamic logic. Platzer (Platzer, 2010) also proposed to introduce differential equations in the program with their respective domain constraints or invariants. Programs are used inside the modalities as they determine the course of action of the cyber-physical system. The actions in $d\mathcal{L}$ are described by the following grammar

$$\alpha, \beta \ni x := \theta \mid x' = \theta \,\&\, \chi \mid ?\chi \mid \alpha \cup \beta \mid \alpha; \beta \mid \alpha^*$$

The first two cases denote, respectively, the discrete and continuous transitions, while the *test* case $?\chi$ blocks all transitions in which χ does not hold. The last three cases are for composition of programs: non–deterministic choice, sequential composition and non–deterministic iteration.

Observe that these regular expressions define programs as usual. In particular the usual imperative operators can be easily defined over there. For instance

- if χ then α fi $\equiv (?\chi; \alpha) \cup (?\neg\chi)$
- if χ then α else β fi $\equiv (?\chi; \alpha) \cup (?\neg\chi; \beta)$

Actually, KeYmaera uses some syntactic sugar from imperative languages to expresses hybrid programs (HP). The above if_then_else operator is an example of a common abbreviation.

As a side note, loop invariants, denoted as $\alpha^*@invariant(F)$, are necessary. The expression annotates a loop α with a recommendation to use formula F as a loop invariant for the loop induction proof rule.

$d\mathcal{L}$ formulas are given by the grammar

$$\phi, \psi \ni \theta_1 = \theta_2 \mid \theta_1 \leq \theta_2 \mid \neg\phi \mid \phi \wedge \psi \mid \phi \rightarrow \psi \mid [\alpha]\phi$$

The $[\alpha]$ operator, the box modality, allows to refer to all transitions of a given a program α. $[\alpha]\phi$ means "at the end of all transitions by α, ϕ holds" (safety property). With the implication it is possible to express conditions like

$$\phi \rightarrow [\alpha]\psi$$

Meaning that "if ϕ holds then, at the end of all transitions by α, ϕ holds". This expression resembles Hoare triples, a well-known logic for validation of imperative programs

$$\phi\{\alpha\}\psi$$

Dually, there is also the diamond modality, $\langle\alpha\rangle$, can be expressed by negation of the box modality

$$\langle\alpha\rangle\psi \leftrightarrow \neg[\alpha]\neg\psi$$

This allows to establish liveness properties of the HP α.

Table 3 collects typical $d\mathcal{L}$ expressions and their informal meaning .

$d\mathcal{L}$	Operator	Meaning
$\theta_1 \sim \theta_2$	comparison	The formula is true iff $\theta_1 \sim \theta_2$ with $\sim \in \{=, >, <, \geq, \leq\}$
$\neg\phi$	negation or not	The formula is true if ϕ is false
$\phi \vee \psi$	conjunction or and	The formula is true if both ϕ and ψ are true
$\phi \wedge \psi$	disjunction or "or"	The formula is true if ϕ is true or if *psi* is true
$\phi \rightarrow \psi$	implication or implies	The formula is true if ϕ is false or ψ is true
$\phi \leftrightarrow \psi$	bi-implication or equivalent	The formula is true if both ϕ and ψ are true or false
$\forall_x \phi$	universal quantifier	The formula is true if ϕ is true for all values of the variable x
$\exists_x \phi$	existential quantifier	The formula is true if ϕ is true for some values of the variable x
$[\alpha]\phi$	Box modality	The formula is true if ϕ is true after all runs α of HP
$\langle\alpha\rangle\phi$	Diamond modality	The formula is true if ϕ is true after at least one run α of HP

Table 3.: $d\mathcal{L}$ operators and their informal meaning.

2.3 FORMALISATION OF PLC PROGRAMS

The term formalisation in Computer Science is increasingly more popular, nowadays. The IEC-61131-3 standard is no exception. The two reasons for the formalisation of the PLC programs come from the increasing strictness of safety and quality of requirements, which entail the need for formal verification, validation or/and simulation and formal analyses. The second reason is the necessity to have a formal representation of these programs due to the constant evolution of hardware and this may lead to the obsolescence of existing PLC programs. The absence of a formal representation could therefore imply a total re-implementation of the program (Younis and Frey, 2003).

PLC programs can be specified at three different levels:

- Parts of the control program (algorithms): This formalisation does not require a full description of the language elements of the PLC, since it only focuses on the algorithms and not on the complete programs. This approach is useful whenever a precise function of a controller must be tested or transferred to another system.
- The complete programs: At this level of formalisation, a model of the program behaviour is derived. After obtaining the model, it can be tested using various test methods of verification

and validation. This is particularly useful because a fully platform-dependent program can be re-implemented and adapted on new hardware.

- The whole control configurations: Important formalisation for re-implementation of control systems software on new control systems hardware. This formalisation formalises several PLC programs on one or more controllers.

This dissertation focuses on verification and validation, a kind of formal technique which helps in enforcing safety, liveness and timing properties. Let us now review of some formalisms proposed in the literature for each description level/language in PLC programs:

- Programming language ST:
  - In (Canet, 2001), the different construct and variables of the language are modelled using communicating automata. These automata are composed which, usually, results in a huge model. After optimisation they can be evaluated by a processor of SMV models.
- Programming language IL:
  - The formalisation described on (Sreenivas and Krogh, 1991) uses Condition/Event systems to model the overall structure of the PLC. The use model-checker VERDICT to verify properties of the composition of the models together with the given model of the system is covered by (Kowalewski et al., 1999).
  - Automata can be used to model PLC algorithms which are programmed in a subset of IL which includes timers of type Time on Timer (TON) (modelled as timed automata). However complex elements of the language, for instance function and function block calls, are not considered in this approach. The type of variables is also restricted to the boolean type, (Mader and Wupper, 1999) With this approach, a tool was developed which translate IL program to timed automaton (Willems, 1999). This tool allows the use of integer variables in addiction to the Boolean variables and the resulting model can be verified by the UPPAAL model checker (Larsen et al., 1995).
  - The use of abstract interpretation also is possible. This enables static analysis of IL programs. This method allows static checking for possible run-time errors and it provides information about the program structure which can be useful to pinpoint dead code or infinite loops (Bornot et al., 2000b).
  - In (Canet et al., 2000), a method for translating IL programs into transition system is considered. The behavioural properties of the controlled systems are modelled as LTL and the operational semantics are coded to SMV for use in a model-checker.
  - Petri nets are also used to model IL programs as reported in (Mertke and Menzel, 2000).
- Programming language SFC:

  - The approach used in (Bornot et al., 2000a) is a translation of the SFC synstax into SMV model checking source code. The generated SMV is then used to verify causal dependencies between input variables and reachability properties (Mcmillan, 1992).
  - Timed automata are also used to formalise PLC program written in SFC (Kowalewski et al., 1998). This approach uses the tool Hytech to prove the validity of the model (Henzinger et al., 1997).
  - The conversion of SFC to Hybrid Automata is described in (Hassapis G., 1998). The resulting model is then used to check for reachability problems. This method is the foundation of the tool presented in this dissertation, which is based from an adaptation from (Nellen and Ábrahám, 2012).
  - The formalisation in (Brinksma and Mader, 2000) is part of a case study for the verification of hybrid systems (Mader et al., 2001), which aims at verifying and designing a PLC program for an experimental chemical plant. Promela/SPIN (Holzmann, 1997) is used to the verification of the PLC program and also to obtain time optimal schedules.
  - In (Roussel and Lesage, 1996), state machines are used to represent SFC programs.

- Programming language LD:
  - Algorithms in LD can be translated to state automata. (Rossi and Schnoebelen, 2000) offers a method for an automated verification of LD and timed function blocks of type Time On Timer (TON). SMV is used as a symbolic model check to check properties.
  - The LD programs can be translated to complementary-places Petri Nets (I Hatono and Tamura, 1996).

- Programming language FBD:
  - A toolset named PLCTOOLS was introduced by (Baresi et al., 2000) to model FBD programs as high level timed petri nets (Ghezzi et al., 1991). MATLAB and SIMULINK enables suitable means for specifying and simulating the model.
  - In (Glück and Krebs, 2015) Kleene algebra is used to create an interactive verification for PLC programs mainly focused in FBD using the KIV system (Reif, 1992).

3

SFC TO KEYMAERA: THE APPROACH

As discussed in the first chapter, the focus of this dissertation is the validation of SFC program using the KEYMAERA theorem prover. Therefore, this is the core chapter where this approach is explained in its different stages of the translation of a subset of the SFC programming language to KEYMAERA. The result is a tool[1] that connects a SFC program int a KEYMAERA model built using HASKELL as a programming language. The aim of this tool is to facilitate the act of proving safety properties of SFC programs using KEYMAERA.

The chapter covers the formalisation of the SFC language and how to add the differential equations to the program. In a second stage, the formal transformation of the SFC to hybrid automata is detailed as well as a brief explanation of how the tool implements this transformation is exposed. Lastly, the chapter explains how the resulting hybrid automata is translated to a KEYMAERA hybrid program. Figure 3 show cases all the steps from the SFC language to KEYMAERA. Each of these steps are described with further detail in this chapter.

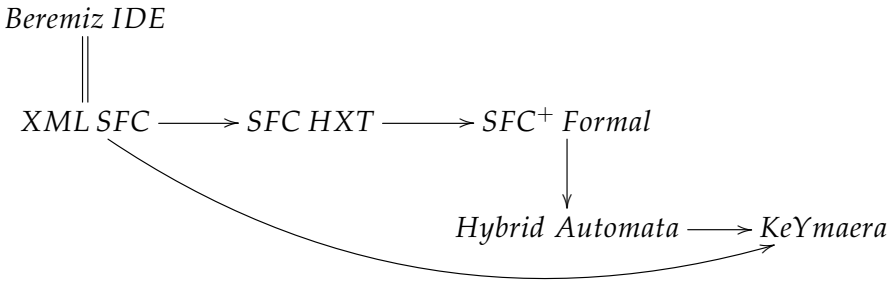


Figure 3.: Transformation of SFC XML to KEYMAERA.

3.1 SEQUENTIAL FUNCTION CHARTS PLUS DIFFERENTIAL EQUATIONS

Reference (Nellen and Ábrahám, 2012) proposes an extension to the PLC language sequential function charts entitled *hybrid sequential function charts* (HSFC) which enables the formal verification of the PLC program through adding, to each step, the continuous dynamics of the system. To be more specific, to each step of the controller, a set of conditional ordinary differential equations (CODE) is

1 Available at: https://github.com/Yoanribeiro/SFCToKeYmaera

added which models the behaviour of the continuous dynamics when the controller is in the specific step in which the condition holds.

Firstly, it is necessary to define the formal syntax of the SFC language and fix the semantics since "the semantics of SFCs given in IEC 61131-3 is ambiguous and erroneous, and PLC programming tool developers interpret the ambiguities posed by the standard in different ways, which results in a variety of different tool semantics" (Bauer et al., 2004).

3.1.1 *SFC Syntax*

The SFC syntax is defined by a set of variables which is the union of the input, output and local variables of the program. Then, there is a set of steps and, connecting them, there is a set of guarded transitions. A particular element of set S is initially active, *i.e.* the initial step. A particular transition is taken if and only if the source step is active and the guard evaluates to true, this triggers immediately[2] the inactivation of the source step and activation of the target step. Let G_V be the set of guards over the variables, which evaluates to true or false whenever the values of the variables are updated. There is also a kind of transition which connects multiple sources/target which are the parallel branching. Each step has a set of action blocks which indicates what must be performed upon activation. An action block $b = (q, a)$ is a pair with a qualifier q and an action a. The qualifier specifies when the action must be performed, $q \in \{entry, do, exit\}$. In the IEC standard $entry$ corresponds to the qualifier $P1$ which is executed only the first time the step is activated, do represents the N qualifier which is performed every time the step is activated and the exit qualifier is the $P0$ which is executed at the deactivation of the step. There are other qualifiers which can be formally represented, however, in this dissertation, they will not be considered. Similarly, an action a can only be a variable assignment, even if it can be another SFC program. The history flag determines if the SFC program has memory, in the case it does, in each execution cycle, the step which is activated is the step that was active in the last cycle, otherwise, each execution cycle starts with its initial step. Formally,

Definition 2 (SFC syntax)**.** *A SFC program consists of a tuple*

$$C = (V, S, A, s_0, T, block, \sqsubset, \prec, Hist)$$

where

- *V is a finite set of variables;*
- *S is a finite set of steps;*
- *A is a finite set of actions referring to variables in V in assignment and to SFCs whose variable and actions sets are susbsets of V and A respectively,*
- *$s_0 \in S$: represents the initial step;*

2 Whenever the controller activates

- $T \subseteq (2^S \setminus \{\emptyset\}) \times G_V \times (2^S \setminus \{\emptyset\})$ *is a finite set of transitions with multiple target/source which define the begin/end of parallel branching;*
- $block : S \to 2^{B_A}$ *is a function which assigns a set of action blocks to each step;*
- $\sqsubset \subseteq A \times A$ *is a total order on actions;*
- $\prec \subseteq T \times T$ *is a partial order on transitions;*
- $Hist \in 0, 1$ *is a history flag: if equal to 1 then the SFC program has active history.*

3.1.2 *SFC semantics*

Programs running on a PLC follows these steps cyclically:

1. Checks the environment and obtains the input data which triggers the update of the values of the variables.
2. Verifies the transitions that have to be taken and executes them.
3. Executes the actions accordingly in their priority order.
4. Sends the data to the output variable.

There is a time delay between two PLC cycles, this can be different for every other cycle, however, the delay is delimited by a lower bound δ_t and an upper bound δ_u. The first and last step of the PLC cycle define the communication with the environment; the remaining steps require the definition of states and configurations. For a given SFC program $E = (V, S, A, s_0, T, block, \sqsubset, \prec, Hist)$, with D denoting the union of all type domains, a *state* $\sigma \in \Sigma$ of E is a function $\sigma : V \to D$ which assigns a value from its respective domain to each variable $v \in V$. The state transformation is also a function $f : \Sigma \to \Sigma$. Then, a configuration is defined as $(\sigma, readyS, activeS, activeA) \in \Sigma \times S \times S \times A$: the state of the SFC, the set of ready steps; the set of active steps, the set of active actions which is sorted by the $\prec$ relation.

$Config$ defines the set of configurations and the initial configuration of an SFC is defined by $(\sigma_0, \{s_0\}, \emptyset, \emptyset)$. For a SFC E and the *configuration* $e = (\sigma, readyS, activeS, activeA)$ of E the sets of enabled and taken transition are defined as:

$$
\begin{aligned}
enabled(E, e) &= \{\, (s, g, s') \in T \mid s \subseteq activeS \wedge c \models g \,\} \\
taken(E, e) &= \{\, t = (s, g, s') \in enabled(E, e) \mid \forall_{t_1 = (s_1, g_1, s_1) \in enabled(E,e)} \bullet s \cap s_1 = \emptyset \vee t_1 \prec t \,\}
\end{aligned}
$$

This semantics focuses on computation steps inbetween the first and last step of the PLC cycle: computation steps start with a configuration $(\sigma, readyS, activeS, activeA)$ where σ was already

updated with new input data from the environment. The computation of the new configuration $(\sigma', readyS', activeS', activeA')$ is the result of executing the steps 2 and 3 of the PLC cycle. And lastly, the output is sent to the environment. If the source step of all taken transitions is replaced by the target step of those transitions afterwards, the new set of ready states is obtained. The new set of active steps and active actions are then recursively calculated by the function $computeActiveSets$.

Definition 3 (SFC Semantics)**.** *Let S be a SFC without actions with nested SFC. The transition relation $\rightarrow \subseteq Config \times Config$ relates configurations*

$$(\sigma, readyS, activeS, activeA) \rightarrow (\sigma', readyS', activeS', activeA')$$

as follows:

- $readyS' = (readyS \setminus source(taken(C,c)) \cup target(taken(C,c))$,
- $(activeS', unsortedActiveA') = computeActiveSets(readyS', \emptyset, \emptyset, C, c, ActivA)$
- $activeA' = sort(unsortedActiveA', \sqsubset)$ *and*
- $\sigma' = (a_1 \ldots . a_m)(\sigma)$ *where* $activeA' = a_m \ldots . a_1$

The definition of the $computeActiveSets$ can be found in the paper (Nellen and Ábrahám, 2012) and all definition of the semantics and syntax are all excellently described and explained in (Bauer et al., 2004).

3.1.3 *Conditional ordinary differential equations*

Conditional differential equation, CODE, are the ODEs which models the behaviour of the environment in which the programmable logic controller is. In (Nellen and Ábrahám, 2012) a Conditional ODE is defined:

Definition 4 (Conditional ODE System)**.** *Let ODE_{V_c} be the set of all ordinary differential equations over V_c (V_c is the set of continuous variables), and Conds, the set of all conditions. A conditional ODE system consists of a pair $(ODEs, cond)$ where $cond \in Conds$ and $ODEs \subseteq ODE_{V_c}$. The set of all conditional ODE system over V_c is denoted by $CODE_{V_c}$*

An example would be a CODE that models the increasing of temperature t until a specific value $maxTemp$ is reached

$$(t' = 2\ , t \leq maxTemp)$$
$$(t' = 0\ , t = maxTemp)$$

3.2 BEREMIZ SFC TO HASKELL SFC

After programming a SFC program in the IDE Beremiz, the program can be compiled to C code using the Matiec compiler, however what unifies the majority of the IDEs used by the IEC standard is the XML representation given by PLCOpen[3]. It enables the exchange of programs, libraries, project between IDEs. Therefore, the tool uses as input the XML file which contains the POU to verify.

3.2.1 *SFCHXT*

The component of the tool that is responsible for parsing the input resorts to the Haskell XML Toolbox (HXT) library. The tool is responsible for parsing the file and obtaining the SFC structure in the form of a Haskell data record. Firstly, however, it is created an intermediate structure called SFCHXT (Appendix B) which is basically the representation of the XML constructs in Haskell, thus simplifying the parsing process. After obtained, the SFCHXT structure, is transformed by the tool into its formal SFC structure (Appendix C) to proceed to the translation to an hybrid automaton.

Firstly, in order to produce the SFCHXT structure, the parser locates all the program organisation units (POUs) which are coded using the SFC language. In XML, it is structured as follows

```
<pous>
  <pou name="name" pouType="type">
    <interface>
    ...
    </interface>
    <body>
      <SFC>
      ...
      </SFC>
    </body>
  </pou>
</pous>
```

The XML structure allows an easy distinction between the POUs, grouping them in the *pous* tag. Descending in the branch of the *pou* tag, there are two groups, the *interface* which has all the variables and, the *body*, where the code of the program is located. From here, the tool obtains POUs whose body is programmed in *SFC* and it proceeds in obtaining the name of the program, thus giving a name to the resulting model. The input, output and local variables can either be initialised or not, in XML

```
<localVars>
  <variable name="A">
    <type>
```

3 PLCOpen XML Website: http://www.plcopen.org/pages/tc6_xml/xml_intro/index.htm

```
      <TYPE1/>
    </type>
  </variable>
  <initialValue>
    <simpleValue value="val"/>
  </initialValue>
</localVars>
<inputVars>
  <variable name="B">
    <type>
      <TYPE2/>
    </type>
  </variable>
</inputVars>
<outputVars>
  <variable name="C">
    <type>
      <TYPE3/>
    </type>
  </variable>
</outputVars>
```

The type of the variable must be restricted, since the variables used in hybrid automata and KEYMAERA are real numbers. This implies that the types allowed are integers, reals and booleans.

After obtaining the variables, the tool starts exploring the content of the *SFC* tag, since these elements are inside the *body* tag the type created was a *BodyElem* which is described in the Appendix B The first step is to obtain the steps of the program. In XML, these are structured as

```
<step localId="id" name="nameStep" initialStep="value" height="x" width="y">
  <connectionPointIn>
    <connection refLocalId="refId"/>
  </connectionPointIn>
</step>
```

The tool creates a *BodyElem* with type step which stores the *localId* if the step is a target the *refId* corresponds to the component that is connected to this step.

After collecting all the steps, the actions are the next component of the SFC program to be collected by the tool, these are structured as follows

```
<actionBlock localId="id" height="valHeight" width="valWidth">
  <position x="valx0" y="valy0"/>
  <connectionPointIn>
```

```
    <relPosition x="valx1" y="valx1"/>
    <connection refLocalId="refId">
      ...
    </connection>
  </connectionPointIn>
  <action localId="refId1" qualifier="qual">
    <relPosition x="valx2" y="valy2"/>
    <inline>
      <ST>
        <xhtml:p><![CDATA[action]]></xhtml:p>
      </ST>
    </inline>
  </action>
</actionBlock>
```

Each action block has the *refLocalId* referring to the step where this action block is executed. The *action* tag has the instruction which has to be performed and, the attribute qualifier, informs whether the action is to be executed the first time the step is activated ($P1$), every time the step is activated (N), or when this step is exited ($P0$).

Finally, the last component of the SFC to be dealt with are the transitions. These can be grouped in three categories: the simple transitions, the divergence transitions and the convergence transition.

The simple transition is the component that contains the condition and a specific source and target. The divergence selection and the convergence selection allows respectively a source to have multiple targets and a target to have multiple sources.

```
<transition localId="id" height="valHeight" width="valWidth">
  <position x="valx0" y="valy0"/>
  <connectionPointIn>
    <relPosition x="valx1" y="valy1"/>
    <connection refLocalId="refID">
    ...
    </connection>
  </connectionPointIn>
  <connectionPointOut>
    ...
  </connectionPointOut>
  <condition>
    <inline name="">
      <ST>
        <xhtml:p><![CDATA[cond]]></xhtml:p>
      </ST>
    </inline>
  </condition>
</transition>
```

The important information to extract from the *transition* is its own identifier, the *refLocalId* which refers to its source and the condition in which the transition is taken.

```
<selectionConvergence localId="id" height="valHeight" width="valWidth">
  <position x="valx1" y="valy1"/>
  <connectionPointIn>
    <relPosition x="valx1" y="valy1"/>
    <connection refLocalId="refId1">
        ...
    </connection>
  </connectionPointIn>
  <connectionPointIn>
    <relPosition x="val2x" y="valy2"/>
    <connection refLocalId="refId2">
      ...
    </connection>
    </connectionPointIn>
  <connectionPointOut>
    ...
  </connectionPointOut>
</selectionConvergence>
```

In the *selectionConvergence* tag, the identifier of the component must be stored and all the *refLocalId* of all the connected transitions. In the example they are *refId1* and *refId2*.

```
<selectionDivergence localId="id" height="valHeight" width="valWidth">
  <position x="valx0" y="valy0"/>
    <connectionPointIn>
    <relPosition x="valx1" y="valy1"/>
    <connection refLocalId="refId">
      ...
    </connection>
  </connectionPointIn>
  <connectionPointOut formalParameter="">
    <relPosition x="valx2" y="valy2"/>
  </connectionPointOut>
  <connectionPointOut formalParameter="">
    <relPosition x="valx3" y="valy3"/>
  </connectionPointOut>
</selectionDivergence>
```

Similarly, in the *selectionDivergence*, the identifier of the component and the *refLocalId* must be stored, since the first will be referred in a number of transitions which diverge from this component and the second is the source of all of those transitions.

The last important component is the comments since they contain the annotations that enrich the SFC program.

```
<comment localId="id" height="valHeight" width="valWidth">
  <position x="valx" y="valy"/>
  <content>
    <xhtml:p><![CDATA[annotation]]></xhtml:p>
  </content>
</comment>
```

After parsing the XML structure, the direct representation of the SFC is created, the tool transforms the SFCHXT structure into its formal representation, in Haskell (see Appendix C).

3.3 SFC ANNOTATION

When the representation of SFC in XML is translated to the Haskell representation of the language, the next step is to parse the annotations. There are five kinds of annotations that extend the SFC language in Beremiz IDE. Note that not all the annotations are necessary to create a model from the program, however it may require some handmade tweaks in the resulting model for it to run properly in KEYMAERA.

1. *CODE Annotation* : The name is self-explanatory, for each step the CODEs are annotated.

2. *Init Annotation* : The pre-conditions of program can be stated in this annotation.

3. *Invariant Annotation* : The expression that is invariant in every execution of the program should be stated in this annotations.

4. *ToProve Annotation* : The expression stated here represents the post-condition of the program or the property to prove.

5. *Extra Annotation* : These annotation are basically a shortcut that can be used to swap expressions that will not be useful once the program is transformed in its model by an equivalent expression.

The conditional ordinary differential equations of a step are annotated using the following grammar

$$\begin{aligned} CODES &::= ODES\ \text{':'}\ Cond\ CODES' \\ CODES' &::= \epsilon \\ &\quad |\ \text{';'}\ CODES \\ \text{with } ODES &::= Exp\ ODES'\ , \\ ODES' &::= \epsilon \\ &\quad |\ \text{','}\ ODES \\ \text{and } Cond &::= BExp \end{aligned}$$

where ϵ represents the empty string, Exp are differential equations, *i.e.* equation with primed variables and $BExp$ are boolean expressions. After parsing the resulting structure is a set of pairs $(ODE, condition)$ which is the definition of a $CODE$.

Then the CODE Annotation is defined as

$$\begin{aligned} CODEAnnot &::= \text{'@'}\ nameStep\text{"->"}CODES\ CODEAnnot' \\ CODEAnnot' &::= \epsilon \\ &\quad |\ \text{'\textbackslash n'}CODEAnnot \end{aligned}$$

Example: @step->$x' = 1 : x \leq m$

The aim of this annotation is to create a set of pairs $(nameStep, CODES)$ and which is then used to access the $CODES$ by $nameStep$. The function $accessCODES : Step \rightarrow CODES$ is defined.

The triple formed by pre-condition, invariant and post-condition are defined by

$$\begin{aligned} initAnnot &::= \text{'@init:'}\ BExp \\ invAnnot &::= \text{'@invariant: '}\ \text{BExp} \\ toProveAnnot &::= \text{'@toProve: '}\ \text{BExp} \end{aligned}$$

Resulting in three expression to be stored in the structure of the hybrid automaton.

Finally the extra annotations are defined as follows

$$
\begin{aligned}
extraAnnot &::= \text{'@Extras:\n'}\ swapBExp\ extraAnnot' \\
extraAnnot' &::= \epsilon \\
&\quad \mid swapBExp\ extraAnnot' \\
swapBExp &::= BExp\ \text{':'}\ BExp
\end{aligned}
$$

This results in set of pair $(oldExp, newExp)$.

3.4 FROM SFC ANNOTATED TO HYBRID AUTOMATA

After defining the annotation language, the next step is the transformation of the annotated SFC to Hybrid automata. This process has two steps which are based in Hybrid SFC (Nellen and Ábrahám, 2012).

1. Transform the pure SFC program into Hybrid Automata;
2. Adapt HA of step 1 with the CODEs resulting in the final HA.

As already stated in the previous section, CODEs are annotated in the SFC program using comments of the Beremiz IDE. Moreover, there are other types of annotation which are relevant for the resulting model in KeYmaera.

3.4.1 *From SFC to Hybrid Automaton*

The SFC transformation to hybrid automaton consists of the following elements:

1. Each step of the SFC program corresponds to a location of the hybrid automaton.
2. The initial state of the HA is the initial step.
3. A clock variable is introduced, denominated t, which specifies the duration between two PLC cycles; This implies that t must be bounded $\delta_l \leq t \leq \delta_u$.
4. An invariant $t \leq \delta_u$ is added to every location to force the automaton to leave the location at time δ_u at the latest.
5. No transition can be taken, before δ_l; then the guard $t \geq \delta_l$ is added to all the transitions.
6. When a transition is taken the clock variable is reset.

7. For each transition in the program, another is created in the automaton, the resulting transition connects the source and target location of the corresponding source and target steps.

8. The actions are sorted according to the action order $\sqsubset$.

9. To model the execution of the *do* action, a self-loop is added to all locations.

10. The guard for the self-loop is the conjunction of the negation of all the guards outgoing from the step.

11. The guards of the transitions are defined to ensure the order defined by $\prec$ taking the guard of the transition and negating the conjunction of guards of higher priority.

Formally the transformation is:

Definition 5 (Transformation of SFC to Hybrid Automaton)**.** *Let*

$$C = (V, S, A, s_0, T, block, \sqsubset, \prec, Hist)$$

be the SFC program without nested components and parallel transitions. The program C is transformed into an hybrid automaton

$$H = (Loc, Var_H, Edge, Act, Inv, Init)$$

as follows:

- $L = S$,
- $Var_H = V \cup \{t\}$,
- $Edge = \cup_{s \in E_s}$ *for each* $s \in S$ *with outgoing transitions* $t_1, \ldots, t_n \in T$ *ordered by the* $\prec$ *order,* $E_s = (\bigcup_{i=1}^{n} (s, \mu_i, s_i)) \cup (s, \mu_s, s)$ *with*
  - μ_i *is the set of valuation pairs* (v, v^*) *with* $v \models t \geq \delta_l \wedge g_i \wedge \bigwedge_{j=i+1}^{n} \neg g_j$ *and* v^* *results from* v *by applying in decreasing priority order the exit actions of* s*, the entry and do actions of* s_i *and* $t := 0$.
  - μ_s *is the set of all valuation pairs* (v, v^*) *with* $v \models t \geq \delta_l \wedge \bigwedge_{j=1}^{n} \neg g_j$ *and* v^* *results from* v *by applying in decreasing priority order the do action of* s *and* $t := 0$.
- *Act function* $\forall_{s \in L} \bullet Act(s) = t' = 1$ *and* $\forall_{v \in (L \setminus \{t\})} \bullet Act(s) = v' = 0$
- *Inv function is defined by* $\forall_{s \in L} \bullet Inv(s) = t \leq \delta_u$
- $Init = \{(s_0, v_0)\}$ *the first element is the initial step and* v_0 *the initial values of the variables.*

The resulting translation can be seen in Figure 4.

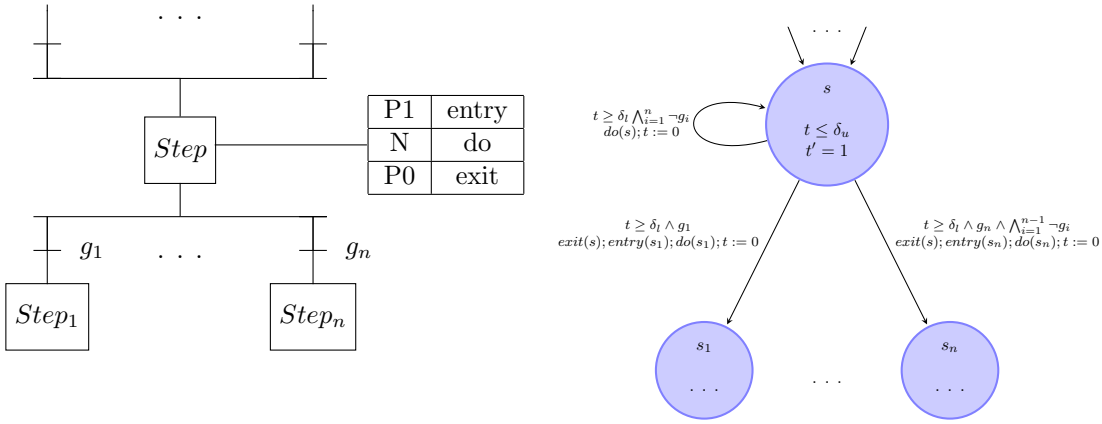


Figure 4.: Transformation SFC to Hybrid Automaton.

3.4.2 *From SFCs and CODES to Hybrid Automaton*

Once the SFC is translated to an hybrid automaton, the next stage of the translation is to create the final hybrid automaton. Firstly, the set CODES is obtained from parsing the annotations $SCODES$ and its respective function $accessCODES$. Then for each step, $s \in S$ with CODEs $\{(ODE_1, c_1), \ldots, (ODE_n, c_n)\}$, the location resulting from the first transformation is replaced by $n+1$ copies $s_1, \ldots, s_{n+1}$; the invariant of each of these copies $i = 1, \ldots, n$ is defined by $c_i \wedge \bigwedge_{j=1}^{i-1} \neg c_j$; the ODE of each of the copies is given by ODE_i. To the invariant of the $n+1$ copy is added the expression $\bigwedge_{j=1}^{n} \neg c_j$. There must be two transitions between each two copies s_i and s_j, $i \neq j$, to enable the switching from one copy to another when the evaluation of the conditions changes, the results can be seen in Figure 5

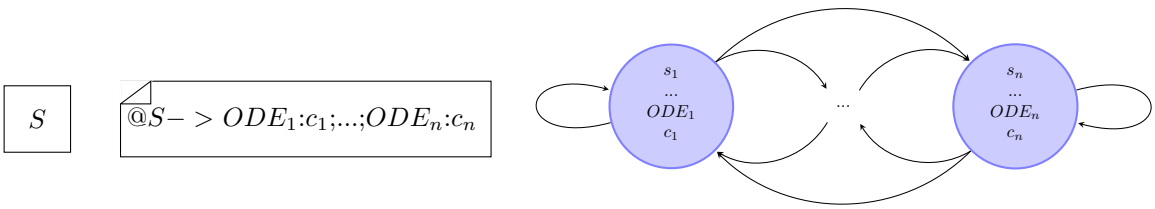


Figure 5.: Adaptation of annotated step to Hybrid automaton.

Definition 6 (Transformation of SFC with CODEs to Hybrid Automaton)**.** *Let*

$$C = (V, S, A, s_0, T, block, \sqsubset, \prec, Hist)$$

be a SFC program without nested components and parallel transitions. The program C is transformed into an hybrid automaton

$$H = (Loc, Var_H, Edges, Act, Inv, Init)$$

together with a set of $CODES = \{(S, CODES)\}$*, the respective function accessCODES and the set of continuous variables* V_c*, defined as follows:*

- $Loc = \bigcup_{s \in S} L_s$ *with* $L_s = \{s_i \mid 1 \leq i \leq (\#(accessCODES(s)) + 1)\}$,
- $Var_H = V \cup V_c \cup \{t\}$,
- *For each step* $s \in S$ *with* $accessCODES(s) = \{(ODE_1, cond_1), \ldots, (ODE_m, cond_m)\}$ *and outgoing transitions* $t_1, \ldots, t_n \in T$ *ordered by* $\prec$ $E =$
  - *For each* $s_k \in L_s$ *there is an edge* (s_k, μ, s_k) *with* μ *the set of all* (v, v^*) *with* $v \models t \geq \delta_l \wedge \bigwedge_{j=1}^{n} \neg g_j$ *and* v^* *the result from v by applying the ordered sequence of do actions of s plus* $t := 0$.
  - *For all* $s_k, s_l \in L_S, s_k \neq s_l$ *there are edges* (s_k, μ, s_l) *and* (s_l, μ, s_k) *with* μ *the identity relation.*
- *Act function* $\forall_{s \in S}, accessStep(s) = \{(ODE_1, cond_1), \ldots, (ODE_m, cond_m)\} \wedge L_s = \{s_1, \ldots, s_{m+1}\} \bullet$ $Act(s_i) = ODE_i \cup t' = 1, i = 1, \ldots, m$ *and* $Act(s_{m+1}) = t' = 1$
- *Inv function* $\forall_{s \in S}, accessStep(s) = \{(ODE_1, cond_1), \ldots, (ODE_m, cond_m)\} \wedge L_s = \{s_1, \ldots, s_{m+1}\} \bullet$ $Inv(s_i) = cond_i \wedge (\bigwedge_{j=1}^{i-1} \neg cond_j) \wedge t \leq \delta_u, i = 1, \ldots, m$ *and* $Inv(s_{m+1}) = (\bigwedge_{j=1}^{m} \neg cond_j) \wedge t \leq \delta_u$
- $Init = \{(s_0, v_0)\}$ *the first element is the initial step and* v_0 *the initial values of the variables.*

3.5 HYBRID AUTOMATA TO KEYMAERA

The final transformation that must be done to conclude the transformation is to represent the HA in a hybrid program for KEYMAERA. This is easily done since a hybrid automata can be represented naturally as a hybrid program once the second is a representation of the first (Platzer, 2010).

Considering the HA $H = (V, L, E, I, F, init)$ and the $initCond, toProveandcycleInv$. For the resulting HP, all the variables of V are declared in the $programVariable$ environment together with a variable st representing the state in the automaton; the delays of SFC are represented as $functions$ environment. The model is defined as follows:

$$(initCond \wedge 0 < \delta_l \wedge 0 < \delta_u \wedge \delta_l < \delta_u)[initVariables](st = init \rightarrow [\phi](postCondition))$$

The *initCond* and *postCondition* are obtained by the annotation. The *initVariables* is basically the attribution of the value in which each variables is initialised. The $st = initState$ is an abuse of notation since the init is a location $init \in L$ a $st \in \mathbb{R}$. The ϕ program is defined as:

$$
\begin{aligned}
&\forall_{l \in L \wedge e_1,\ldots,e_m \in \{e' \mid source(e)=l\}} \bullet \\
&\qquad ?(st = l); \\
&\qquad\qquad ?(cond(e_1)); \\
&\qquad\qquad event(e_1); \\
&\qquad\qquad st := target(e_1) \\
&\qquad\qquad \cup \\
&\qquad\qquad \ldots \\
&\qquad\qquad \cup \\
&\qquad\qquad ?(cond(e_n)); \\
&\qquad\qquad event(e_n); \\
&\qquad\qquad st := target(e_n) \\
&\qquad\qquad \cup \\
&\qquad\qquad \{flow(l), inv(l)\}
\end{aligned}
$$

The functions *source* and *target* return the source and target of the transitions respectively; the function *cond* is the function that, given an edge, returns the condition for the transitions to occur. The *event* function gives back the set of assignments to be executed when the transitions is taken.

4

CASE STUDY: THE WATER HEATING TANK

In Chapter 3, it is shown how the transformation from sequential function charts with annotations to KEYMAERA was designed in order to implement a tool to prove safety properties of a SFC program. The main aim of this chapter is to understand what is the actual extent of the implemented tool with the support of a case study.

Though the water tank problem is a fairly usual (e.g (Platzer, 2010)) example used in the domain of hybrid systems, it can be modified to a water heating tank, to further increment the complexity of the problem. Firstly, the description of the problem is exposed, then the PLC program is designed and, finally, the tool is used to obtain the model and prove some properties.

4.1 DESCRIPTION OF THE PROBLEM

Consider an empty tank. The goal of this system is to heat the temperature of the water to be used in some other task. The tank has to be fully filled and only then the controller starts the heating process. After a while, the tank is again emptied and the process starts over.

To implement this tank, it must have a sensor which detects that it is empty giving a *Empty* signal. However, the process only begins once the *Start* signal is given. Once both signals are received the discharge valve is closed and the filling pump is activated. When the sensor detects that the tank is full, *Full* signal is sent and the filling pump is stopped. Consequently, the heater is switched on. When the sensors reads 90 degrees Celsius, the heater is switched off and a timer is set to 20 minutes. Once the timer reaches 20 minutes, the controller opens the discharge valve and begin the emptying process. Finally, when the water-tank is empty, the process begins anew.

The filling process begins and its dynamics is described by the differential equations that give the height of the water, namely, $y' = 3$. When the heater is on the differential equations that describes its behaviour is give by $temp' = 4$ knowing that the temperature of the water starts at 20 degrees Celsius, and finally, once the controller opens the discharge valve the height of the water is given by the equation $y' = -4$.

The safety property that the program must guarantee throughout the all execution of the PLC program is that the water level is always between *Empty* and *Full* signals.

4.2 THE SFC PROGRAM AND THE GENERATED MODEL

4.2.1 *Program and annotations*

From the description of the problem, the type of variables we need to declare can be easily identified.

1. The signal variables, which must be of type boolean, to represent whether a sensor or a other controller is activated or not.
2. The quantified variables, of type real ($\mathbb{R}$), represent a "sensor" which outputs real values.

The signal variables are *Empty*, *Start*, *Full*, the discharge valve (*dv*), the pump and the heater. The quantified variables are the timer tm' and the temperature $temp$.

The steps of the program are deduced from the states of the system. Therefore for this specific program the steps are:

- Step1 represents the empty tank;
- Step2 represents the filling process;
- Step3 represents the heating process;
- Step4 represents the waiting process;
- Step5 represents the emptying process.

With respect to the actions of this program, every action has, as quantifier, P1 since they are all executed only at the first time the step is activated. There is no actions to be executed on Step1; on Step2 the discharge variable evaluates to $false$ and the pump to $true$; on Step3 the pump variable evaluates to $false$ and the heater variable to $true$; the actions in Step4 are the deactivation of the heater variable and the assignment to the timer 20 min; finally the last step activates discharge variable.

The transitions are the last components to be defined:

- From Step1 to Step2 $Empty = True \wedge Start = True$ must hold.
- From Step2 to Step3 $Full = True$ must hold.
- From Step3 to Step4 $temp = 90$ must hold.
- From Step4 to Step5 $tm = 20$ must hold.
- From Step5 to Step1 $Empty = True$ must hold.

Formally, SFC water heating tank program is illustrated in Figure 6. The water heating tank consists of the SFC $C = (V, S, A, s_0, T, block, \sqsubset, \prec, Hist)$ with

- $V = \{Empty, Start, Full, pump, dv, heater, temp, tm\}$;
- $S = \{Step1, Step2, Step3, Step4, Step5\}$;
- $A = \{dv := False, pump := True, pump := False, heater := True, heater := False, tm := 0, dv := True\}$. The set is ordered by relation $\sqsubset$
- $s_0 = Step1$;
- $T = \{(Step1, Empty = True \wedge Start = True, Step2), (Step2, Full = Ture, Step3), (Step3, temp = 90, Step4), (Step4, tm = 20, Step5), (Step5, Empty = True, Step1)\}$;
- block $= \{(Step2, \{dv := False, pump := True\}), (Step3, \{pump := False, heater := True\}), (Step4, \{heater := False, tm := 20\}), (Step5, \{dv := True\})\}$. The set is ordered by $\prec$;
- $Hist = 1$.

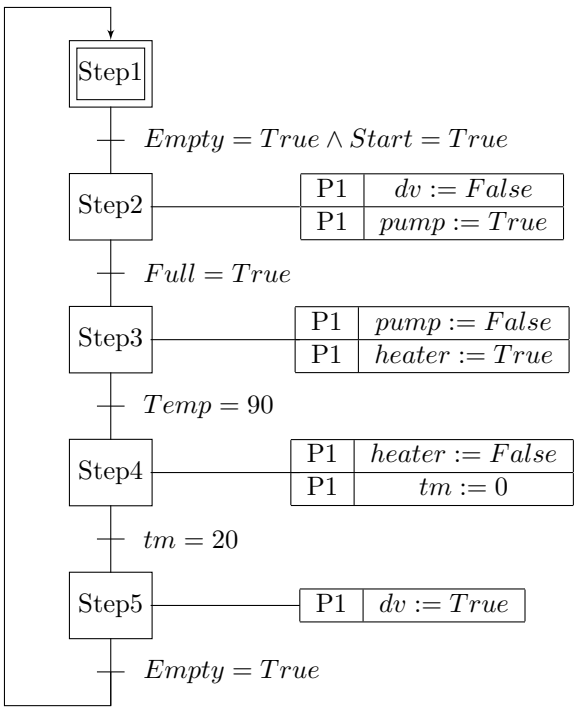


Figure 6.: SFC program of the water heating tank problem.

Once the program is designed, as shown in Figure 6, the next stage is to annotate it. The description of the problem already provides the differential equations and also the conditions in which they must hold. Then on Step2 has $y' = 1$ as differential equation and condition $y <= maxH$. At Step3 the

system dynamics is given by the set of ODEs $y' = 0$ and $temp' = 2$ and the $temp \leq 90$ must hold for the second ODE. Step4 has $tm' = 1$ and $temp' = 0$ as ODES describing its dynamics and the condition $tm \leq 20$ must be true for the first equation. At Step5 $tm' = 0$ and $y' = -2$ describes the dynamics of the system and $y \geq 0$ must be true for the second differential equation.

The precondition, invariant and postconditions are, respectively, $maxH > 0 \wedge 0 <= y <= maxH$. The remaining conditions are $0 <= y <= maxH$. Note that, for the extra annotation, it is necessary to locate the conditions which are not coherent once the program is translated. These conditions are $Full = True$ and $Empty = True$, consequently these must be substituted by $y = maxH$ and $y = 0$ respectively. The use of $maxH$ introduces an abstract ceiling for the height of the water, since the maximum height was not in the description.

The resulting program in Beremiz can seen in figure 7

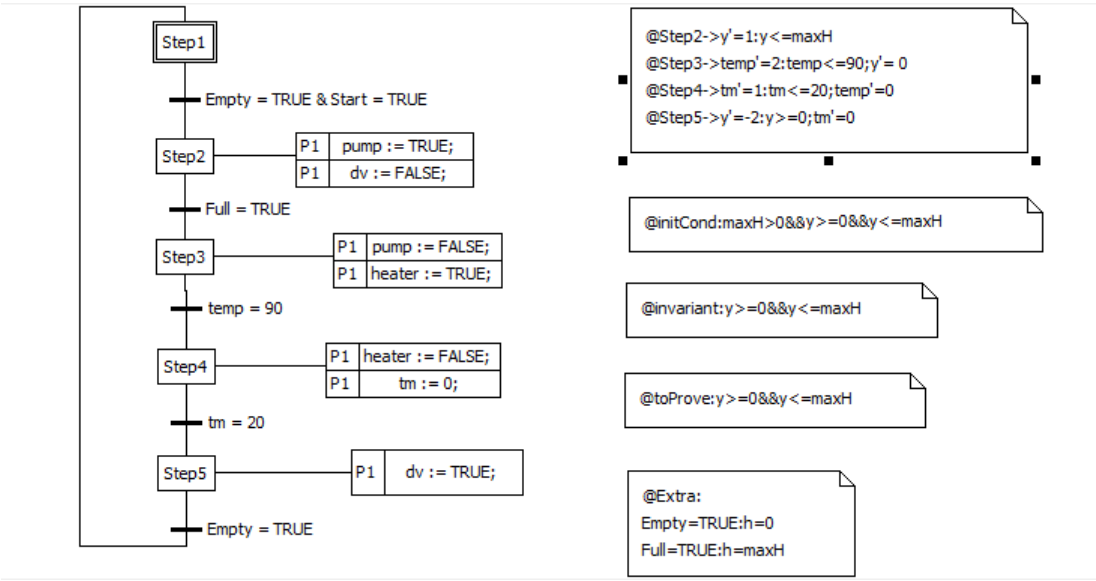


Figure 7.: Water heating tank program with annotation in Beremiz IDE.

The resulting XML file (see appendix E) is then given to the tool to generate the model and the tool returns the message seen on Figure 8

Figure 8.: The resulting message given by the tool after processing the PLC program.

4.2.2 *Model obtained*

The resulting model is quite large since the program has five steps and four of them have two CODE each. Thus,

```
\functions{
  R deltaCycleMin;
  R deltaCycleMax;
}
\programVariables {
  R Empty;
  R Full;
  R Start;
  R dv;
  R heater;
  R maxH;
  R pump;
  R t;
  R temp;
  R tm;
  R y;
  R st;
}
/**
  Step2:0
  Step20:1
  Step3:2
  Step30:3
  Step31:4
  Step4:5
  Step40:6
  Step41:7
  Step5:8
  Step50:9
  Step51:10
  Step1:11
**/
\problem {
  (
    (
      maxH>0.0&y>=0.0&y<=maxH&
      deltaCycleMin>0 & deltaCycleMax>0
          &
      deltaCycleMin < deltaCycleMax
    )
    ->
\[
  t:=0.0;
  st:=11.0
\](
  st=11
  ->
  \[
    (
      (
        ?(st=0);
          (
            ?(y=maxH);
              (
                pump := 0;heater :=
                    1;
                st:=2
              )                ++
            ?(y=maxH);
              (
                pump := 0;heater :=
                    1;
                st:=3
              )                ++
            ?(y=maxH);
              (
                pump := 0;heater :=
                    1;
                st:=4
              )                ++
            ?(!(y=maxH));
              (

                st:=0
              )                ++
            (
                st:=1
            )                  ++
            {t'=1,t<=deltaCycleMax
                ,!(y<=maxH)}
          )
      )
      ++
```

```
(
  ?(st=1);
    (
      ?(y=maxH);
        (
          pump := 0;heater :=
              1;
          st:=2
        )                ++
      ?(y=maxH);
        (
          pump := 0;heater :=
              1;
          st:=3
        )                ++
      ?(y=maxH);
        (
          pump := 0;heater :=
              1;
          st:=4
        )                ++
      ?(!(y=maxH));
        (

          st:=1
        )                ++
      (
          st:=0
      )                ++
      {y'=1.0,t'=1,y<=maxH,t<=
          deltaCycleMax}
    )
)
++
(
  ?(st=2);
    (
      ?(temp=90);
        (
          heater := 0;tm := 0;
          st:=5
        )                ++
      ?(temp=90);
        (
          heater := 0;tm := 0;
          st:=6
        )                ++
      ?(temp=90);
        (
          heater := 0;tm := 0;
          st:=7
        )                ++
      ?(!(temp=90));
        (

          st:=2
        )                ++
      (
          st:=3
      )                ++
      (
          st:=4
      )                ++
      {t'=1,t<=deltaCycleMax
          ,!(temp<=90.0)}
    )
)
++
(
  ?(st=3);
    (
      ?(temp=90);
        (
          heater := 0;tm := 0;
          st:=5
        )                ++
      ?(temp=90);
        (
          heater := 0;tm := 0;
          st:=6
        )                ++
      ?(temp=90);
        (
          heater := 0;tm := 0;
          st:=7
        )                ++
      ?(!(temp=90));
        (

          st:=3
        )                ++
      (
          st:=2
      )                ++
```

```
        (
            st:=4
        )                ++
        {temp'=2.0,t'=1,temp
            <=90.0,t<=
            deltaCycleMax}
    )
)
++
(
  ?(st=4);
    (
      ?(temp=90);
        (
          heater := 0;tm := 0;
          st:=5
        )                ++
      ?(temp=90);
        (
          heater := 0;tm := 0;
          st:=6
        )                ++
      ?(temp=90);
        (
          heater := 0;tm := 0;
          st:=7
        )                ++
      ?(!(temp=90));
        (

          st:=4
        )                ++
      (
          st:=2
      )                ++
      (
          st:=3
      )                ++
      {y'=0.0,t'=1,t<=
          deltaCycleMax,!(temp
          <=90.0)}
    )
)
++
(
  ?(st=5);
    (
      ?(tm=20);
        (
          dv := 1;
          st:=8
        )                ++
      ?(tm=20);
        (
          dv := 1;
          st:=9
        )                ++
      ?(tm=20);
        (
          dv := 1;
          st:=10
        )                ++
      ?(!(tm=20));
        (

          st:=5
        )                ++
      (
          st:=6
      )                ++
      (
          st:=7
      )                ++
      {t'=1,t<=deltaCycleMax
          ,!(tm<=20.0)}
    )
)
++
(
  ?(st=6);
    (
      ?(tm=20);
        (
          dv := 1;
          st:=8
        )                ++
      ?(tm=20);
        (
          dv := 1;
          st:=9
        )                ++
      ?(tm=20);
        (
          dv := 1;
```

```
            st:=10
          )                ++
        ?(!(tm=20));
          (

            st:=6
          )                ++
        (
            st:=5
        )                  ++
        (
            st:=7
        )                  ++
        {tm'=1.0,t'=1,tm<=20.0,t
            <=deltaCycleMax}
    )
)
++
(
  ?(st=7);
    (
      ?(tm=20);
        (
          dv := 1;
          st:=8
        )                  ++
      ?(tm=20);
        (
          dv := 1;
          st:=9
        )                  ++
      ?(tm=20);
        (
          dv := 1;
          st:=10
        )                  ++
      ?(!(tm=20));
        (

          st:=7
        )                  ++
        (
          st:=5
        )                  ++
        (
          st:=6
        )                  ++
        {temp'=0.0,t'=1,t<=
            deltaCycleMax,!(tm
            <=20.0)}
    )
)
++
(
  ?(st=8);
    (
      ?(y=0.0);
        (

          st:=11
        )                  ++
      ?(!(y=0.0));
        (

          st:=8
        )                  ++
        (
          st:=9
        )                  ++
        (
          st:=10
        )                  ++
        {t'=1,t<=deltaCycleMax
          ,!(y>=0.0)}
    )
)
++
(
  ?(st=9);
    (
      ?(y=0.0);
        (

          st:=11
        )                  ++
      ?(!(y=0.0));
        (

          st:=9
        )                  ++
        (
          st:=8
        )                  ++
        (
```

```
            st:=10
        )                ++
        {y'=-2.0,t'=1,y>=0.0,t<=
            deltaCycleMax}
    )
)
++
(
  ?(st=10);
    (
      ?(y=0.0);
        (

          st:=11
        )                ++
      ?(!(y=0.0));
        (

          st:=10
        )                ++
      (
          st:=8
      )                  ++
      (
          st:=9
      )                  ++
      {tm'=0.0,t'=1,t<=
          deltaCycleMax,!(y
          >=0.0)}
    )
)
++
```

```
(
  ?(st=11);
    (
      ?(y=0.0&Start=1);
        (
          pump := 1;dv := 0;
          st:=0
        )                ++
      ?(y=0.0&Start=1);
        (
          pump := 1;dv := 0;
          st:=1
        )                ++
      ?(!(y=0.0&Start=1));
        (

          st:=11
        )                ++
      {t'=1,t<=deltaCycleMax}
    )
)       )*@invariant(
            y>=0.0&y<=maxH
        )
\](
  y>=0.0&y<=maxH
  )
)
}
```

4.3 VERIFYING SAFETY PROPERTIES

The safety property to be proved in the model is that the height of the water has to remain between *Empty* signal and *Full* signal *i.e.* $y \geq 0.0 \wedge y \leq maxH$. KEYMAERA proves it, as shown in Figure 9.

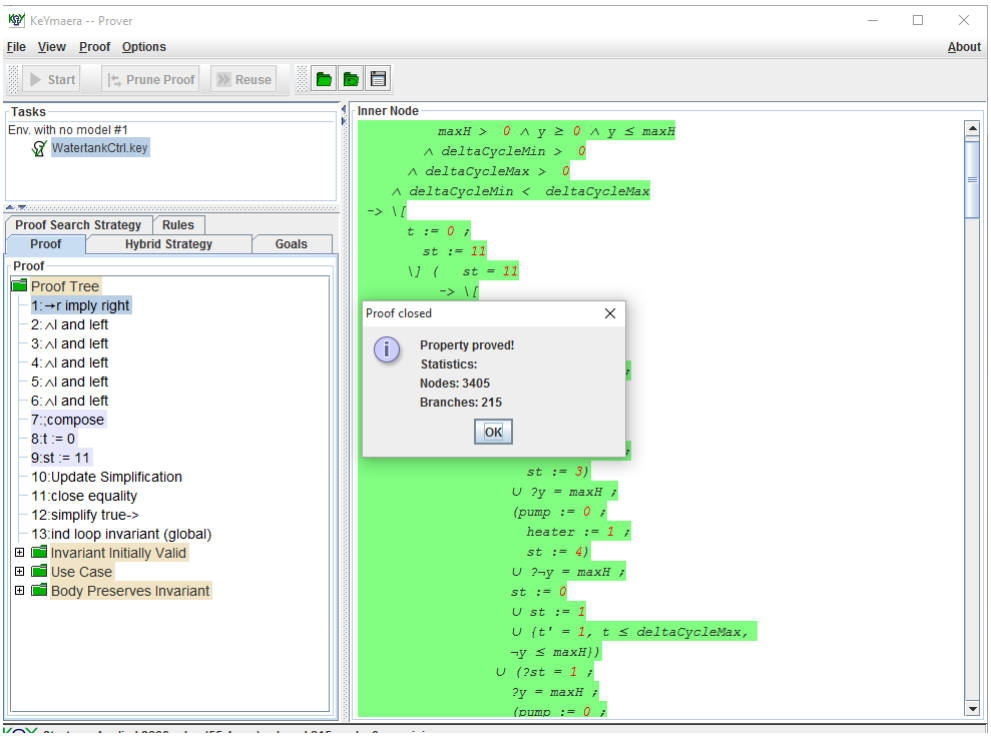


Figure 9.: KEYMAERA window with the proof success message.

However, the model can prove other properties, with the proper updates made in the annotation, namely the *toProve* annotation, which, obviously, must be altered if the property to prove changes, and, consequently, the *initCond* and *invariant* annotations also must be changed. For example, for the property: the temperature is waver between 20 and 90 degrees Celsius the conditions $temp >= 0 \wedge temp <= 90$ must be added to every annotation. KEYMAERA also proves this condition as shown in Figure 10.

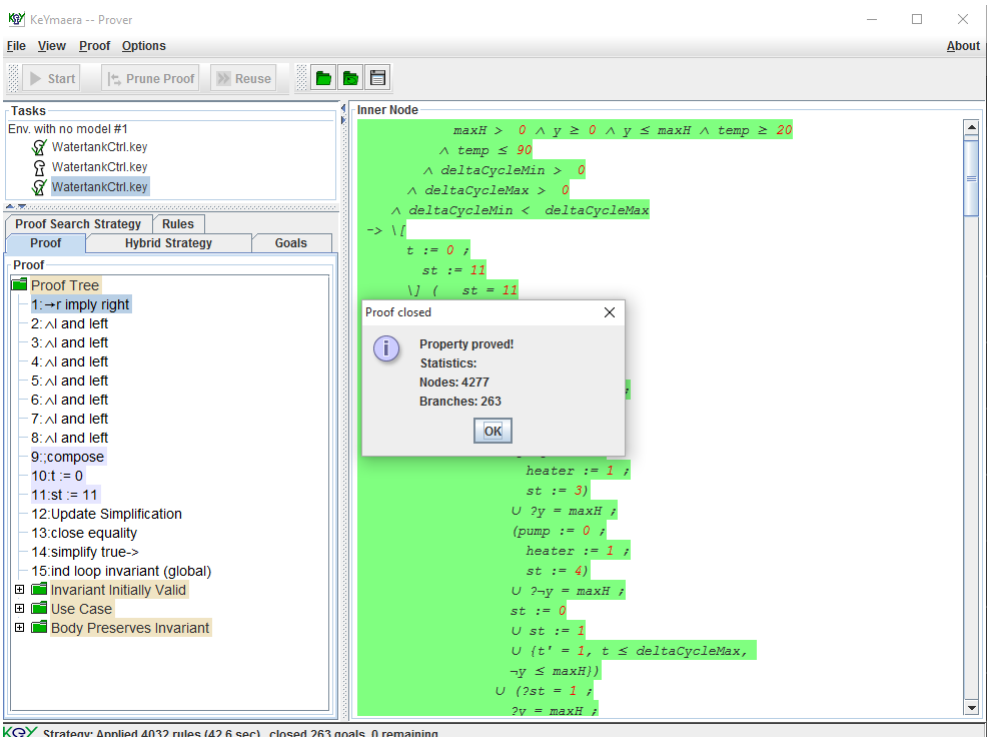


Figure 10.: KEYMAERA window with the proof success message with the temperature condition.

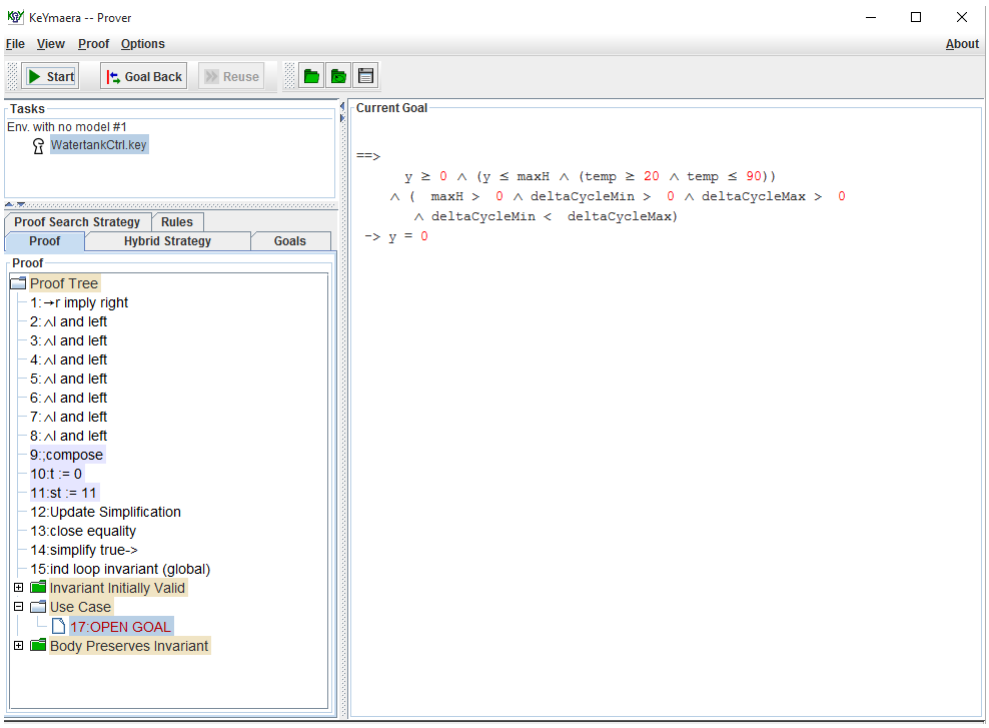


Figure 11.: KEYMAERA window on an unsuccessful attempt to prove $y = 0$.

Furthermore, consider how to prove that the model is not stationary, *i.e.* that the variables do not always have the same value even with the correct precondition and invariant. Considering the preconditions and invariant from the last model, the model cannot prove that the height of the water is always zero. The condition does not hold because the invariant of the program does not imply that the height is always equal to zero (see Figure 11). This allows to conclude that the model does not prove the safety condition trivially, thus proving that the model is not stationary.

5

CASE STUDY:THE PH PLANT

This chapter discusses another case study on the use of KEYMAERA to verify SFC programs. The case study is about a pH plant which is responsible by monitoring the pH of the water in a tank whilst the latter is being filled.

Similarly to Chapter 4, the problem is first discribed in natural language. Then it is scrutinised to obtain the different components to design the desired PLC program. Finally the tool is used to obtain a corresponding hybrid program on top of which some properties of the controller can be proved.

5.1 DESCRIPTION OF THE PROBLEM

Let us consider a water tank that is being filled. The aim of the controller is to monitor the pH level of the water until the tank is completely full. The water pH can vary from normal, to low or high.

The controller must have a *pH sensor*, which provides the current value of the water pH, and a sensor, which gives a *full* signal to halt the filling process. The controller can send signals to valves which are responsible for introducing a treatment to decrease or increase the value of pH, the controller can also send a signal to the *pump* to halt the filling process.

When acting to decrease the water pH, the controller state is given by the differential equation $ph' = -0.7$, which is active while the condition $ph > 7.0$ holds. When increasing the pH level, this value is given by the equation $ph' = 0.9$ and the treatment continues while the condition $pH < 7.0$ holds. When the water has a neutral pH and since it is filling (causing an increase of the pH level) the dynamics of the pH is given by the equation $ph' = 0.1$ and, the pH level must stay between 6.8 and 7.2. The filling process dynamics is the same for all levels of pH and is given by the differential equation $y' = 0.1$, with the water height between 0 and 1 ($0 \leq y < 1$).

The property which ensures the tank system is safe is that the tank does not overflow and the pH level is always between 6.8 and 7.2

5.2 THE SFC PROGRAM AND THE GENERATED MODEL

5.2.1 *Program and annotations*

After exposing the problem, the next stage is to extract the necessary components to design the SFC program. Similarly to the water tank case study discussed in the previous chapter, there are two type of variables, namely, the signal variables and the quantified variables. The first are the *full* signal given when the water height maximum is reached, the *pump* signal responsible by the valve which fills the tank, and the two valves to pump the treatment to decrease the pH level, *vphm*, or to increase it, the *vphp*. The quantified variable is *ph* representing the sensor that shows the pH value.

The different steps of the program are the following:

- Step1 represents the water tank with neutral pH;
- Step2 represents the water tank with high pH (alkaline);
- Step3 represents the water tank with low pH (acidic);
- Step4 represents the full water tank.

The qualifier of every action in the PLC program is P1, entailing that they must be executed only at the first time the step is activated. At Step1, both valves responsible for the treatment are closed, *i.e.* the program sends the signal false to both variables. At Step2, the valve used to decrease the pH level is opened, *i.e.* the program sends a true signal to the corresponding variable. At Step3, the valve for increasing of the water pH is opened, *i.e.* true signal is sent to the corresponding variable. Finally at Step4, the pump variable is set to false to stop the filling process.

Finally, the last component of this SFC program is the set of transitions:

- From Step1 to Step2 $ph \geq 7.2$ must hold.
- From Step1 to Step3 $ph \leq 6.8$ must hold.
- From Step1 to Step4 $full = True$ must hold.
- From Step2 to Step4 $full = True$ must hold.
- From Step2 to Step1 $ph \leq 7.0$ must hold.
- From Step3 to Step4 $full = True$ must hold.
- From Step3 to Step1 $ph \geq 7.0$ must hold.

The SFC program is graphically represented in Figure 12.

Formally, it consists of the tuple $C = (V, S, A, s_0, T, block, \sqsubset, \prec, Hist)$, where

- $V = \{full, vphp, vphm, pump, ph\}$;
- $S = \{Step1, Step2, Step3, Step4\}$;
- $A = \{vphp := False, vphm := False, vphm := True, vphp := True, pump := False\}$ The set is ordered by relation $\sqsubset$
- $s_0 = Step1$;
- $T = \{(Step1, ph \geq 7.2, Step2), (Step1, ph \leq 6.8, Step3), (Step1, full = True, Step4), (Step2, full = True, Step4), (Step2, ph \leq 7.0, Step1), (Step3, full = True, Step4), (Step3, ph \geq 7.0, Step1)\}$;
- block $= \{(Step1, \{vphp := False, vphm := False\}), (Step2, \{vphm := True\}), (Step3, \{vphp := True\}), (Step4, \{pump := False\})\}$. The set is ordered by relation $\prec$;
- $Hist = 1$.

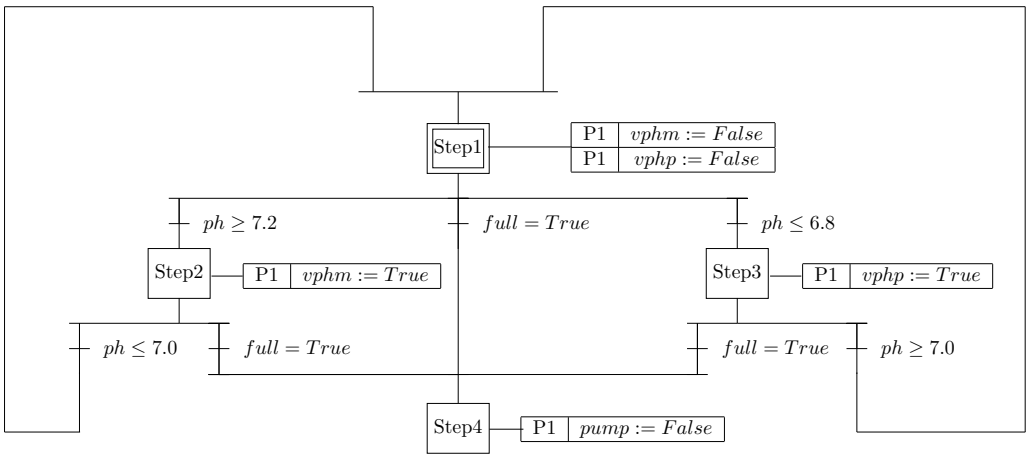


Figure 12.: SFC program representing the pH plant problem.

After designing the SFC program, the step part of the process to obtain corresponding hybrid model of the pH plant is to annotate the PLC program. Starting with the conditional differential equations annotation, from Step1 to Step4 the equation which represents the dynamics of the water height is $y' = 0.1$ and the condition $0 \leq y < 1$ must hold. Then for Step1 the differential equation for the dynamics of the pH level is given by $ph' = 0.1$ and the condition $6.8 < ph < 7.2$ must hold. For Step2 the differential equation $ph' = -0.7$ and the condition $ph \geq 7.0$ must hold. For Step3 the equation $ph' = 0.9$ and the condition $ph \leq 7.0$ must hold. Finally, for Step4 the relevant differential equations are $ph' = 0$ and $y' = 0$.

The preconditions, invariants and postconditions are, respectively, $0 \leq y \leq 0.1 \wedge 7.05 \leq ph \leq 7.15$ and the remaining two conditions are $0 \leq y \leq 1 \wedge 6.8 \leq ph \leq 7.2$. For the extra annotation, it

is necessary to locate the conditions which are not coherent once the program is translated. There is only one being $full = True$, it must be replaced by $y = 1$.

The program, which can be seen in Figure 13, results in the XML description (see appendix F) is then given to the tool to generate the KEYMAERA hybrid program.

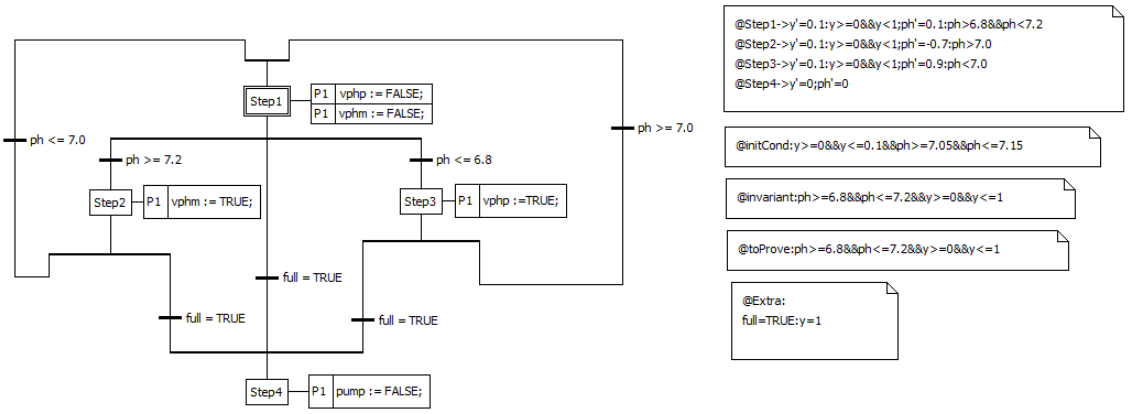


Figure 13.: pH plant program with annotation in Beremiz IDE.

5.2.2 *Model obtained*

The resulting hybrid program is:

```
\functions{
  R deltaCycleMin;
  R deltaCycleMax;
}
\programVariables {
  R full;
  R ph;
  R pump;
  R t;
  R vphm;
  R vphp;
  R y;
  R st;
}
/**
  Step1:0
  Step10:1
  Step11:2
  Step2:3
  Step20:4
  Step21:5
  Step3:6
  Step30:7
  Step31:8
  Step4:9
  Step40:10
  Step41:11
**/
\problem {
  (
    (
      y>=0.0&y<=0.1&ph>=7.05&ph<=7.15&
      deltaCycleMin>0 & deltaCycleMax>0
          &
      deltaCycleMin < deltaCycleMax
    )
    ->
    \[
      t:=0.0;
      st:=0.0
    \](
```

```
st=0
->
\[
  (
    (
      ?(st=0);
        (
          ?(ph>=7.2&!(ph<=6.8)&!(y
              =1.0));
            (
              vphm := 1;
              st:=3
            )                  ++
          ?(ph>=7.2&!(ph<=6.8)&!(y
              =1.0));
            (
              vphm := 1;
              st:=4
            )                  ++
          ?(ph>=7.2&!(ph<=6.8)&!(y
              =1.0));
            (
              vphm := 1;
              st:=5
            )                  ++
          ?(ph<=6.8&!(y=1.0));
            (
              vphp :=1;
              st:=6
            )                  ++
          ?(ph<=6.8&!(y=1.0));
            (
              vphp :=1;
              st:=7
            )                  ++
          ?(ph<=6.8&!(y=1.0));
            (
              vphp :=1;
              st:=8
            )                  ++
          ?(y=1.0);
            (
              pump := 0;
              st:=9
            )                  ++
          ?(y=1.0);
            (
              pump := 0;
              st:=10
            )                  ++
          ?(y=1.0);
            (
              pump := 0;
              st:=11
            )                  ++
          ?(!(ph>=7.2)&!(ph<=6.8)
              &!(y=1.0));
            (

              st:=0
            )                  ++
          (
              st:=1
          )                  ++
          (
              st:=2
          )                  ++
          {t'=1,t<=deltaCycleMax
              ,!(y>=0.0&y<1.0),!(
              ph>6.8&ph<7.2)}
      )
  )
  ++
  (
    ?(st=1);
      (
        ?(ph>=7.2&!(ph<=6.8)&!(y
            =1.0));
          (
            vphm := 1;
            st:=3
          )                  ++
        ?(ph>=7.2&!(ph<=6.8)&!(y
            =1.0));
          (
            vphm := 1;
            st:=4
          )                  ++
        ?(ph>=7.2&!(ph<=6.8)&!(y
            =1.0));
          (
            vphm := 1;
            st:=5
          )                  ++
```

```
            ?(ph<=6.8&!(y=1.0));
              (
                vphp :=1;
                st:=6
              )                  ++
            ?(ph<=6.8&!(y=1.0));
              (
                vphp :=1;
                st:=7
              )                  ++
            ?(ph<=6.8&!(y=1.0));
              (
                vphp :=1;
                st:=8
              )                  ++
            ?(y=1.0);
              (
                pump := 0;
                st:=9
              )                  ++
            ?(y=1.0);
              (
                pump := 0;
                st:=10
              )                  ++
            ?(y=1.0);
              (
                pump := 0;
                st:=11
              )                  ++
            ?(!(ph>=7.2)&!(ph<=6.8)
                &!(y=1.0));
              (

                st:=1
              )                  ++
            (
                st:=0
            )                 ++
            (
                st:=2
            )                 ++
            {y'=0.1,t'=1,y>=0.0&y
                <1.0,t<=
                deltaCycleMax}
        )
    )
++
(
  ?(st=2);
    (
      ?(ph>=7.2&!(ph<=6.8)&!(y
          =1.0));
        (
          vphm := 1;
          st:=3
        )                  ++
      ?(ph>=7.2&!(ph<=6.8)&!(y
          =1.0));
        (
          vphm := 1;
          st:=4
        )                  ++
      ?(ph>=7.2&!(ph<=6.8)&!(y
          =1.0));
        (
          vphm := 1;
          st:=5
        )                  ++
      ?(ph<=6.8&!(y=1.0));
        (
          vphp :=1;
          st:=6
        )                  ++
      ?(ph<=6.8&!(y=1.0));
        (
          vphp :=1;
          st:=7
        )                  ++
      ?(ph<=6.8&!(y=1.0));
        (
          vphp :=1;
          st:=8
        )                  ++
      ?(y=1.0);
        (
          pump := 0;
          st:=9
        )                  ++
      ?(y=1.0);
        (
          pump := 0;
          st:=10
        )                  ++
```

```
        ?(y=1.0);
          (
            pump := 0;
            st:=11
          )                  ++
        ?(!(ph>=7.2)&!(ph<=6.8)
            &!(y=1.0));
          (

            st:=2
          )                  ++
        (
            st:=0
        )                  ++
        (
            st:=1
        )                  ++
        {ph'=0.1,t'=1,ph>6.8&ph
            <7.2,t<=
            deltaCycleMax,!(y
            >=0.0&y<1.0)}
    )
)
++
(
  ?(st=3);
    (
      ?(y=1.0&!(ph<=7.0));
        (
          pump := 0;
          st:=9
        )                  ++
      ?(y=1.0&!(ph<=7.0));
        (
          pump := 0;
          st:=10
        )                  ++
      ?(y=1.0&!(ph<=7.0));
        (
          pump := 0;
          st:=11
        )                  ++
      ?(ph<=7.0);
        (
          vphp := 0;vphm := 0;
          st:=0
        )                  ++
      ?(ph<=7.0);
        (
          vphp := 0;vphm := 0;
          st:=1
        )                  ++
      ?(ph<=7.0);
        (
          vphp := 0;vphm := 0;
          st:=2
        )                  ++
      ?(!(y=1.0)&!(ph<=7.0));
        (

          st:=3
        )                  ++
      (
          st:=4
      )                  ++
      (
          st:=5
      )                  ++
      {t'=1,t<=deltaCycleMax
          ,!(y>=0.0&y<1.0),!(
          ph>7.0)}
    )
)
++
(
  ?(st=4);
    (
      ?(y=1.0&!(ph<=7.0));
        (
          pump := 0;
          st:=9
        )                  ++
      ?(y=1.0&!(ph<=7.0));
        (
          pump := 0;
          st:=10
        )                  ++
      ?(y=1.0&!(ph<=7.0));
        (
          pump := 0;
          st:=11
        )                  ++
      ?(ph<=7.0);
        (
```

```
                vphp := 0;vphm := 0;
                st:=0
              )                  ++
            ?(ph<=7.0);
              (
                vphp := 0;vphm := 0;
                st:=1
              )                  ++
            ?(ph<=7.0);
              (
                vphp := 0;vphm := 0;
                st:=2
              )                  ++
            ?(!(y=1.0)&!(ph<=7.0));
              (

                st:=4
              )                  ++
            (
                st:=3
            )                  ++
            (
                st:=5
            )                  ++
            {y'=0.1,t'=1,y>=0.0&y
                <1.0,t<=
                deltaCycleMax}
        )
    )
    ++
    (
      ?(st=5);
        (
          ?(y=1.0&!(ph<=7.0));
            (
              pump := 0;
              st:=9
            )                  ++
          ?(y=1.0&!(ph<=7.0));
            (
              pump := 0;
              st:=10
            )                  ++
          ?(y=1.0&!(ph<=7.0));
            (
              pump := 0;
              st:=11
            )                  ++
          ?(ph<=7.0);
            (
              vphp := 0;vphm := 0;
              st:=0
            )                  ++
          ?(ph<=7.0);
            (
              vphp := 0;vphm := 0;
              st:=1
            )                  ++
          ?(ph<=7.0);
            (
              vphp := 0;vphm := 0;
              st:=2
            )                  ++
          ?(!(y=1.0)&!(ph<=7.0));
            (

              st:=5
            )                  ++
          (
              st:=3
          )                  ++
          (
              st:=4
          )                  ++
          {ph'=-0.7,t'=1,ph>7.0,t
              <=deltaCycleMax,!(y
              >=0.0&y<1.0)}
        )
    )
    ++
    (
      ?(st=6);
        (
          ?(y=1.0&!(ph>=7.0));
            (
              pump := 0;
              st:=9
            )                  ++
          ?(y=1.0&!(ph>=7.0));
            (
              pump := 0;
              st:=10
            )                  ++
          ?(y=1.0&!(ph>=7.0));
```

```
        (
          pump := 0;
          st:=11
        )                ++
      ?(ph>=7.0);
        (
          vphp := 0;vphm := 0;
          st:=0
        )                ++
      ?(ph>=7.0);
        (
          vphp := 0;vphm := 0;
          st:=1
        )                ++
      ?(ph>=7.0);
        (
          vphp := 0;vphm := 0;
          st:=2
        )                ++
      ?(!(y=1.0)&!(ph>=7.0));
        (

          st:=6
        )                ++
      (
          st:=7
      )                  ++
      (
          st:=8
      )                  ++
      {t'=1,t<=deltaCycleMax
          ,!(y>=0.0&y<1.0),!(
          ph<7.0)}
    )
)
++
(
  ?(st=7);
    (
      ?(y=1.0&!(ph>=7.0));
        (
          pump := 0;
          st:=9
        )                ++
      ?(y=1.0&!(ph>=7.0));
        (
          pump := 0;
          st:=10
        )                ++
      ?(y=1.0&!(ph>=7.0));
        (
          pump := 0;
          st:=11
        )                ++
      ?(ph>=7.0);
        (
          vphp := 0;vphm := 0;
          st:=0
        )                ++
      ?(ph>=7.0);
        (
          vphp := 0;vphm := 0;
          st:=1
        )                ++
      ?(ph>=7.0);
        (
          vphp := 0;vphm := 0;
          st:=2
        )                ++
      ?(!(y=1.0)&!(ph>=7.0));
        (

          st:=7
        )                ++
      (
          st:=6
      )                  ++
      (
          st:=8
      )                  ++
      {y'=0.1,t'=1,y>=0.0&y
          <1.0,t<=
          deltaCycleMax}
    )
)
++
(
  ?(st=8);
    (
      ?(y=1.0&!(ph>=7.0));
        (
          pump := 0;
          st:=9
        )                ++
```

```
        ?(y=1.0&!(ph>=7.0));
          (
            pump := 0;
            st:=10
          )                 ++
        ?(y=1.0&!(ph>=7.0));
          (
            pump := 0;
            st:=11
          )                 ++
        ?(ph>=7.0);
          (
            vphp := 0;vphm := 0;
            st:=0
          )                 ++
        ?(ph>=7.0);
          (
            vphp := 0;vphm := 0;
            st:=1
          )                 ++
        ?(ph>=7.0);
          (
            vphp := 0;vphm := 0;
            st:=2
          )                 ++
        ?(!(y=1.0)&!(ph>=7.0));
          (
            st:=8
          )                 ++
        (
            st:=6
        )                 ++
        (
            st:=7
        )                 ++
        {ph'=0.9,t'=1,ph<7.0,t<=
            deltaCycleMax,!(y
            >=0.0&y<1.0)}
    )
)
++
(
  ?(st=9);
    (
      (
          st:=9
        )                 ++
        (
            st:=10
        )                 ++
        (
            st:=11
        )                 ++
        {t'=1,t<=deltaCycleMax}
    )
)
++
(
  ?(st=10);
    (
      (
          st:=10
        )                 ++
        (
            st:=9
        )                 ++
        (
            st:=11
        )                 ++
        {y'=0.0,t'=1,t<=
            deltaCycleMax}
    )
)
++
(
  ?(st=11);
    (
      (
          st:=11
        )                 ++
        (
            st:=9
        )                 ++
        (
            st:=10
        )                 ++
        {ph'=0.0,t'=1,t<=
            deltaCycleMax}
    )
)       )*@invariant(
            ph>=6.8&ph<=7.2&y
                >=0.0&y<=1.0
          )
```

```
      \](                                    }
        ph>=6.8&ph<=7.2&y>=0.0&y<=1.0
        )
    )
)
```

5.3 VERIFYING SAFETY PROPERTY

The safety property to be proved by the model is that the water pH is always between 6.8 and 7.2, *i.e.* $0 \leq y \leq 1 \wedge 6.8 \leq ph \leq 7.2$. The proof in KEYMAERA is shown in Figure 14.

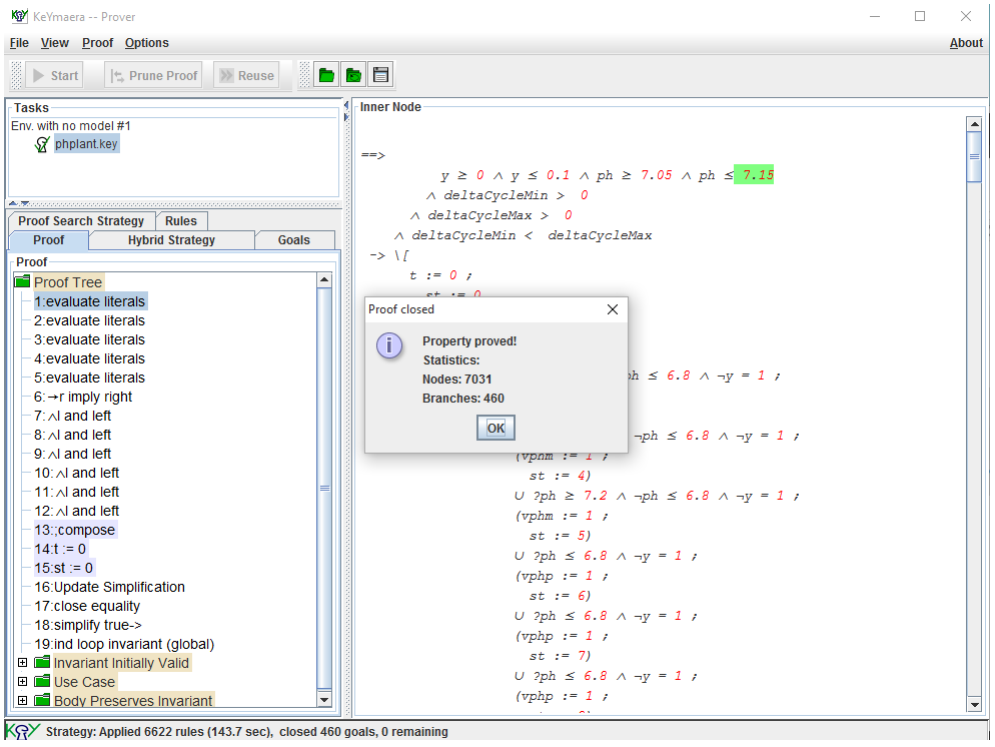


Figure 14.: KEYMAERA window proving the safety property of the pH plant.

6

CONCLUSION AND FUTURE WORK

6.1 ACHIEVEMENTS

PLC programs are paradigmatic in the class of safety critical software. This entails the need for developing the methods of the verification and validation of PLC languages as proposed by the IEC 61131-3. Consequently, these languages have various formalisations. Though, most of these are rooted in the classical techniques of verification from Computer Science, which are focusing on the discrete dynamics of the PLC programs.

This dissertation proposed to verify PLC programs using KEYMAERA theorem prover. This lead to the development of a tool[1] based on the extension of the language SFC proposed in (Nellen and Ábrahám, 2012), hybrid sequential function charts that also provide the translation of HSFC to hybrid automata and on the direct representation of hybrid automata to hybrid program proposed in (Platzer, 2010). Therefore it was possible to obtain a KEYMAERA hybrid program from the each SFC programs (detailed in Section 3.5). Consequently, this method focuses on the interaction between the discrete and continuous dynamics of a system.

To achieve the so called hybrid sequential function charts as in the Beremiz IDE, this dissertation proposed a series of annotations that enrich SFC programs with the conditional ordinary differential equations and other necessary components, namely, the precondition, invariant and postcondition and an extra annotation to trade conditions in the program, which is not translated (detailed in Section 3.3).

6.2 FUTURE WORK

This dissertation can be a starting point to introduce the KEYMAERA theorem prover to the industry as a valid tool to verify and validate safety properties of PLC programs. Furthermore, several lines of work had emerged from the work exposed in this dissertation.

- Implement the translation for the remaining languages in IEC. Evaluate IL, LD and FBD languages and find a way to translate them to KEYMAERA hybrid program.

1 Which is available at: https://github.com/Yoanribeiro/SFCToKeYmaera

- The Structured Text language was the first language used to understand how PLC programs work and its translation is already made to KEYMAERA. The next step is to incorporate the annotations in the language, which entails the need for a full understanding of how the language semantics works (the translation can be seen in Appendix A).
- Development of techniques to refine the hybrid automaton that results from HSFC translation. In Sections 4.2.2 and 5.2.2, the resulting models are large and inelegant. This can consequently lead to an inefficient treatment by KEYMAERA since there is a large amount of states and edges to be considered. Furthermore, the resulting automaton can possibly be simplified to mitigate state explosions in KEYMAERA.
- Add another annotation to enable assignment of continuous variables.

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A

AUXILARY WORK: ST TO KEYMAERA

Keyword	Description	Example	KeYmaera
:=	Assignment	d := 10;	d := 10.
	Call of an FB	`FBName();`	Replacement of the call by its respective code assigning the input variables with the right values

IF	Selection	`IF b THEN a`
		`ELSIF d`
		`THEN c`
		`ELSE d`
		`END_IF`
			`(?b);(a)`
			`++`
			`(?~b && d);c`
			`++`
			`(?~b && ~d);d`

CASE	Multi-selection	**CASE**(b) 1: a 2: b ... **END_CASE;**	(?b=1); a ++ (?b=2); b ++ ...
WHILE	Iteration (2)	**WHILE** n **DO** a **END_WHILE;**	(?b=1); a ++ (?b=2); b ++ ...
REPEAT	Iteration (3)	**REPEAT** a **UNTIL** b **END_REPEAT;**	a;((?~b);a)*;(?b)

Table 4.: ST code to KEYMAERA hybrid program

B

TOOL STRUCTURE: SFCHXT

```
data TypeBodyElem = StepT | ActionBlock | TransitionT | SelectionConvergence |
    SelectionDivergence deriving (Eq, Show)

-- BodyElem (ID, (Type, ( [Reference(ConnIN)], (Qualifier, Text) ) ) )
newtype BodyElem        = BodyElem  { unBodyElem    :: (Int, ( String, (
    TypeBodyElem, ([Int], (String, String) ) ) ) ) } deriving (Eq, Show)
-- VarP (type,(name, ([value], Category) ))
newtype VarP        = VarP      { unVarP    :: (String,(String, ([String], String
    ) )) } deriving (Eq, Show)



data SFCP = SFCP{
          nameP    :: String,
          varsP        :: [VarP],
          stepsP       :: [BodyElem],
          actionsP     :: [BodyElem],
          initStepP    :: BodyElem,
          transitionsP :: [BodyElem],
          annotationsP :: [String],
          histP   :: Bool
        } deriving (Show, Eq)
```

C

TOOL STRUCTURE: SFC

```
-- Step is just the name of the step
newtype Step    = Step      { unStep    :: String } deriving (Eq, Show)
-- Variable has the name and a possible Value and conditions which comes from the
     type of the variable
newtype Var     = Var       { unVar     :: ((String, [Float]), [String]) }
    deriving (Eq, Show)
-- Action has a priority, a quantifier, the step in which it is associate with
    and the action in Structured Text
newtype Action    = Action    { unAction    :: (Int,(String, (Step, String))) }
    deriving (Eq, Show)
-- Transition has a priority, the inicial step and the final step, and the
    condition
newtype Transition  = Transition  { unTransition  :: (Int,((Step,Step),String)) }
     deriving (Eq, Show)


data SFC = SFC {
          name    :: String,
          vars        :: [Var],
          steps       :: [Step],
          actions     :: [Action],
          initStep    :: Step,
          transitions :: [Transition],
          hist    :: Bool,
          annotations :: [String]
        } deriving (Show, Eq)
```

D

TOOL STRUCTURE: HYBRID AUTOMATON

```
-- VarH represents a variable of the HA which can have a inicial value,
    represented as a singleton list of Float, and it also has another singleton
    list which represents extra condition about the variable
newtype VarH     = VarH       { unVarH     :: ((String, [Float]), [String]) }
    deriving (Eq, Show, Ord)
-- Loc is a Node of the HA
newtype Loc      = Loc        { unLoc :: String } deriving (Eq, Show)
-- Edge is self-explanatory ((source,target),(condition,assignments))
newtype Edge = Edge { unEdge :: ((Loc,Loc), (String, String)) } deriving (Eq,
    Show)
-- Invariant of a Loc
newtype Inv = Inv { unInv :: (Loc,String) } deriving (Eq, Show)
-- Differential Equation for a Loc
newtype Flow = Flow { unFlow :: (Loc,String) } deriving (Eq, Show)


-- Map between [Loc] and [Int]
data HybridAutomaton = HA {
                            nameH       :: String,
                            varsH       :: [VarH],
                            locs        :: [Loc],
                            edges       :: [Edge],
                            invs        :: [Inv],
                            flows       :: [Flow],
                            initH       :: Loc,
                            initCond    :: String,
                            toProve     :: String,
                            cycleInv    :: String
                          } deriving (Show, Eq)
```

E

CASE STUDY: WATER HEATING TANK XML

```
<pou name="WatertankCtrl" pouType="functionBlock">
  <interface>
    <localVars>
      <variable name="pump">
        <type>
          <BOOL/>
        </type>
      </variable>
      <variable name="dv">
        <type>
          <BOOL/>
        </type>
      </variable>
    </localVars>
    <inputVars>
      <variable name="Start">
        <type>
          <BOOL/>
        </type>
      </variable>
    </inputVars>
    <outputVars>
      <variable name="heater">
        <type>
          <BOOL/>
        </type>
      </variable>
      <variable name="Full">
        <type>
          <BOOL/>
        </type>
      </variable>
      <variable name="Empty">
        <type>
          <BOOL/>
```

```
        </type>
      </variable>
    </outputVars>
    <localVars>
      <variable name="tm">
        <type>
          <INT/>
        </type>
      </variable>
    </localVars>
    <outputVars>
      <variable name="temp">
        <type>
          <INT/>
        </type>
      </variable>
    </outputVars>
  </interface>
  <body>
    <SFC>
      <step localId="1" name="Step0" initialStep="true" height="27" width="
          42">
        <position x="348" y="31"/>
        <connectionPointIn>
          <relPosition x="21" y="0"/>
          <connection refLocalId="10">
            <position x="369" y="31"/>
            <position x="369" y="21"/>
            <position x="299" y="21"/>
            <position x="299" y="380"/>
            <position x="370" y="380"/>
            <position x="370" y="370"/>
          </connection>
        </connectionPointIn>
        <connectionPointOut formalParameter="">
          <relPosition x="21" y="27"/>
        </connectionPointOut>
      </step>
      <step localId="2" name="Step1" initialStep="false" height="23" width=
          "38">
        <position x="350" y="109"/>
        <connectionPointIn>
          <relPosition x="19" y="0"/>
          <connection refLocalId="6">
            <position x="369" y="109"/>
            <position x="369" y="84"/>
          </connection>
```

```
      </connectionPointIn>
      <connectionPointOut formalParameter="">
        <relPosition x="19" y="23"/>
      </connectionPointOut>
      <connectionPointOutAction formalParameter="">
        <relPosition x="38" y="11"/>
      </connectionPointOutAction>
    </step>
    <step localId="3" name="Step2" initialStep="false" height="23" width=
        "38">
      <position x="350" y="176"/>
      <connectionPointIn>
        <relPosition x="19" y="0"/>
        <connection refLocalId="7">
          <position x="369" y="176"/>
          <position x="369" y="157"/>
        </connection>
      </connectionPointIn>
      <connectionPointOut formalParameter="">
        <relPosition x="19" y="23"/>
      </connectionPointOut>
      <connectionPointOutAction formalParameter="">
        <relPosition x="38" y="11"/>
      </connectionPointOutAction>
    </step>
    <step localId="4" name="Step3" initialStep="false" height="23" width=
        "38">
      <position x="351" y="249"/>
      <connectionPointIn>
        <relPosition x="19" y="0"/>
        <connection refLocalId="8">
          <position x="370" y="249"/>
          <position x="370" y="223"/>
        </connection>
      </connectionPointIn>
      <connectionPointOut formalParameter="">
        <relPosition x="19" y="23"/>
      </connectionPointOut>
      <connectionPointOutAction formalParameter="">
        <relPosition x="38" y="11"/>
      </connectionPointOutAction>
    </step>
    <step localId="5" name="Step4" initialStep="false" height="23" width=
        "38">
      <position x="351" y="320"/>
      <connectionPointIn>
        <relPosition x="19" y="0"/>
```

```
        <connection refLocalId="9">
          <position x="370" y="320"/>
          <position x="370" y="299"/>
        </connection>
      </connectionPointIn>
      <connectionPointOut formalParameter="">
        <relPosition x="19" y="23"/>
      </connectionPointOut>
      <connectionPointOutAction formalParameter="">
        <relPosition x="38" y="11"/>
      </connectionPointOutAction>
    </step>
    <transition localId="6" height="2" width="20">
      <position x="359" y="82"/>
      <connectionPointIn>
        <relPosition x="10" y="0"/>
        <connection refLocalId="1">
          <position x="369" y="82"/>
          <position x="369" y="58"/>
        </connection>
      </connectionPointIn>
      <connectionPointOut>
        <relPosition x="10" y="2"/>
      </connectionPointOut>
      <condition>
        <inline name="">
          <ST>
            <xhtml:p><![CDATA[Empty = TRUE & Start = TRUE]]></xhtml:p>
          </ST>
        </inline>
      </condition>
    </transition>
    <transition localId="7" height="2" width="20">
      <position x="359" y="155"/>
      <connectionPointIn>
        <relPosition x="10" y="0"/>
        <connection refLocalId="2">
          <position x="369" y="155"/>
          <position x="369" y="132"/>
        </connection>
      </connectionPointIn>
      <connectionPointOut>
        <relPosition x="10" y="2"/>
      </connectionPointOut>
      <condition>
        <inline name="">
          <ST>
```

```
        <xhtml:p><![CDATA[Full = TRUE]]></xhtml:p>
      </ST>
    </inline>
  </condition>
</transition>
<transition localId="8" height="2" width="20">
  <position x="360" y="221"/>
  <connectionPointIn>
    <relPosition x="10" y="0"/>
    <connection refLocalId="3">
      <position x="370" y="221"/>
      <position x="370" y="210"/>
      <position x="369" y="210"/>
      <position x="369" y="199"/>
    </connection>
  </connectionPointIn>
  <connectionPointOut>
    <relPosition x="10" y="2"/>
  </connectionPointOut>
  <condition>
    <inline name="">
      <ST>
        <xhtml:p><![CDATA[temp = 90]]></xhtml:p>
      </ST>
    </inline>
  </condition>
</transition>
<transition localId="9" height="2" width="20">
  <position x="360" y="297"/>
  <connectionPointIn>
    <relPosition x="10" y="0"/>
    <connection refLocalId="4">
      <position x="370" y="297"/>
      <position x="370" y="272"/>
    </connection>
  </connectionPointIn>
  <connectionPointOut>
    <relPosition x="10" y="2"/>
  </connectionPointOut>
  <condition>
    <inline name="">
      <ST>
        <xhtml:p><![CDATA[tm = 20]]></xhtml:p>
      </ST>
    </inline>
  </condition>
</transition>
```

```
<transition localId="10" height="2" width="20">
  <position x="360" y="368"/>
  <connectionPointIn>
    <relPosition x="10" y="0"/>
    <connection refLocalId="5">
      <position x="370" y="368"/>
      <position x="370" y="343"/>
    </connection>
  </connectionPointIn>
  <connectionPointOut>
    <relPosition x="10" y="2"/>
  </connectionPointOut>
  <condition>
    <inline name="">
      <ST>
        <xhtml:p><![CDATA[Empty = TRUE]]></xhtml:p>
      </ST>
    </inline>
  </condition>
</transition>
<actionBlock localId="11" height="36" width="118">
  <position x="443" y="105"/>
  <connectionPointIn>
    <relPosition x="0" y="15"/>
    <connection refLocalId="2">
      <position x="443" y="120"/>
      <position x="388" y="120"/>
    </connection>
  </connectionPointIn>
  <action localId="0" qualifier="P1">
    <relPosition x="0" y="0"/>
    <inline>
      <ST>
        <xhtml:p><![CDATA[pump := TRUE;]]></xhtml:p>
      </ST>
    </inline>
  </action>
  <action localId="0" qualifier="P1">
    <relPosition x="0" y="0"/>
    <inline>
      <ST>
        <xhtml:p><![CDATA[dv := FALSE;]]></xhtml:p>
      </ST>
    </inline>
  </action>
</actionBlock>
<actionBlock localId="12" height="36" width="112">
```

```
      <position x="468" y="172"/>
      <connectionPointIn>
        <relPosition x="0" y="15"/>
        <connection refLocalId="3">
          <position x="468" y="187"/>
          <position x="388" y="187"/>
        </connection>
      </connectionPointIn>
      <action localId="0" qualifier="P1">
        <relPosition x="0" y="0"/>
        <inline>
          <ST>
            <xhtml:p><![CDATA[pump := FALSE;]]></xhtml:p>
          </ST>
        </inline>
      </action>
      <action localId="0" qualifier="P1">
        <relPosition x="0" y="0"/>
        <inline>
          <ST>
            <xhtml:p><![CDATA[heater := TRUE;]]></xhtml:p>
          </ST>
        </inline>
      </action>
    </actionBlock>
    <actionBlock localId="13" height="36" width="116">
      <position x="463" y="245"/>
      <connectionPointIn>
        <relPosition x="0" y="15"/>
        <connection refLocalId="4">
          <position x="463" y="260"/>
          <position x="389" y="260"/>
        </connection>
      </connectionPointIn>
      <action localId="0" qualifier="P1">
        <relPosition x="0" y="0"/>
        <inline>
          <ST>
            <xhtml:p><![CDATA[heater := FALSE;]]></xhtml:p>
          </ST>
        </inline>
      </action>
      <action localId="0" qualifier="P1">
        <relPosition x="0" y="0"/>
        <inline>
          <ST>
            <xhtml:p><![CDATA[tm := 0;]]></xhtml:p>
```

```
                </ST>
              </inline>
            </action>
          </actionBlock>
          <actionBlock localId="14" height="30" width="114">
            <position x="480" y="316"/>
            <connectionPointIn>
              <relPosition x="0" y="15"/>
              <connection refLocalId="5">
                <position x="480" y="331"/>
                <position x="389" y="331"/>
              </connection>
            </connectionPointIn>
            <action localId="0" qualifier="P1">
              <relPosition x="0" y="0"/>
              <inline>
                <ST>
                  <xhtml:p><![CDATA[dv := TRUE;]]></xhtml:p>
                </ST>
              </inline>
            </action>
          </actionBlock>
          <comment localId="15" height="70" width="151">
            <position x="636" y="51"/>
            <content>
              <xhtml:p><![CDATA[@Extra:
Empty=TRUE:y=0
Full=TRUE:y=maxH]]></xhtml:p>
            </content>
          </comment>
          <comment localId="16" height="36" width="264">
            <position x="670" y="153"/>
            <content>
              <xhtml:p><![CDATA[@Step2->y'=1:y<=maxH
@Step3->y'= 0;temp'=2:temp<=90
@Step4->tm'=1:tm<=20;temp'=0
@Step5->tm'=0;y'=-2:y>=0]]></xhtml:p>
            </content>
          </comment>
          <comment localId="17" height="36" width="200">
            <position x="707" y="300"/>
            <content>
              <xhtml:p><![CDATA[@toProve:y>=0&&y<=maxH ]]></xhtml:p>
            </content>
          </comment>
          <comment localId="18" height="36" width="207">
            <position x="694" y="237"/>
```

```
        <content>
          <xhtml:p><![CDATA[@invariant:h>=0&&h<=maxH]]></xhtml:p>
        </content>
      </comment>
      <comment localId="19" height="36" width="275">
        <position x="699" y="209"/>
        <content>
          <xhtml:p><![CDATA[@initCond:maxH>0&&y>=0&&y<=maxH]]></xhtml:p>
        </content>
      </comment>
    </SFC>
  </body>
</pou>
```

F

CASE STUDY: PH PLANT XML

```
<pou name="phplant" pouType="functionBlock">
        <interface>
          <outputVars>
            <variable name="pump">
              <type>
                <BOOL/>
              </type>
            </variable>
          </outputVars>
          <inputVars>
            <variable name="full">
              <type>
                <BOOL/>
              </type>
            </variable>
          </inputVars>
          <outputVars>
            <variable name="vphp">
              <type>
                <BOOL/>
              </type>
            </variable>
            <variable name="vphm">
              <type>
                <BOOL/>
              </type>
            </variable>
          </outputVars>
          <inputVars>
            <variable name="ph">
              <type>
                <REAL/>
              </type>
            </variable>
          </inputVars>
```

```
        </interface>
        <body>
          <SFC>
            <comment localId="15" height="70" width="151">
              <position x="782" y="306"/>
              <content>
                <xhtml:p><![CDATA[@Extra:
full=TRUE:y=1]]></xhtml:p>
              </content>
            </comment>
            <comment localId="16" height="96" width="363">
              <position x="775" y="55"/>
              <content>
                <xhtml:p><![CDATA[@Step1->y'=0.1:y>=0&&y<1;ph'=0.1:ph>6.8&&ph<7.2
@Step2->y'=0.1:y>=0&&y<1;ph'=-0.7:ph>7.0
@Step3->y'=0.1:y>=0&&y<1;ph'=0.9:ph<7.0
@Step4->y'=0;ph'=0]]></xhtml:p>
              </content>
            </comment>
            <comment localId="17" height="36" width="310">
              <position x="778" y="259"/>
              <content>
                <xhtml:p><![CDATA[@toProve:ph>=6.8&&ph<=7.2&&y>=0&&y<=1]]></xhtml
                    :p>
              </content>
            </comment>
            <comment localId="18" height="36" width="317">
              <position x="777" y="212"/>
              <content>
                <xhtml:p><![CDATA[@invariant:ph>=6.8&&ph<=7.2&&y>=0&&y<=1]]></
                    xhtml:p>
              </content>
            </comment>
            <comment localId="19" height="36" width="341">
              <position x="776" y="165"/>
              <content>
                <xhtml:p><![CDATA[@initCond:y>=0&&y<=0.1&&ph>=7.05&&ph<=7.15]]></
                    xhtml:p>
              </content>
            </comment>
            <step localId="20" name="Step1" initialStep="true" height="27" width=
                "42">
              <position x="339" y="128"/>
              <connectionPointIn>
                <relPosition x="21" y="0"/>
                <connection refLocalId="31">
                  <position x="360" y="128"/>
```

```
        <position x="360" y="117"/>
        <position x="360" y="117"/>
        <position x="360" y="105"/>
      </connection>
    </connectionPointIn>
    <connectionPointOut formalParameter="">
      <relPosition x="21" y="27"/>
    </connectionPointOut>
    <connectionPointOutAction formalParameter="">
      <relPosition x="42" y="13"/>
    </connectionPointOutAction>
  </step>
  <actionBlock localId="21" height="36" width="110">
    <position x="400" y="126"/>
    <connectionPointIn>
      <relPosition x="0" y="15"/>
      <connection refLocalId="20">
        <position x="400" y="141"/>
        <position x="381" y="141"/>
      </connection>
    </connectionPointIn>
    <action localId="0" qualifier="P1">
      <relPosition x="0" y="0"/>
      <inline>
        <ST>
          <xhtml:p><![CDATA[vphp := FALSE;]]></xhtml:p>
        </ST>
      </inline>
    </action>
    <action localId="0" qualifier="P1">
      <relPosition x="0" y="0"/>
      <inline>
        <ST>
          <xhtml:p><![CDATA[vphm := FALSE;]]></xhtml:p>
        </ST>
      </inline>
    </action>
  </actionBlock>
  <selectionDivergence localId="22" height="1" width="282">
    <position x="218" y="175"/>
    <connectionPointIn>
      <relPosition x="142" y="0"/>
      <connection refLocalId="20">
        <position x="360" y="175"/>
        <position x="360" y="157"/>
        <position x="360" y="157"/>
        <position x="360" y="165"/>
```

```
          <position x="360" y="165"/>
          <position x="360" y="155"/>
        </connection>
      </connectionPointIn>
      <connectionPointOut formalParameter="">
        <relPosition x="0" y="1"/>
      </connectionPointOut>
      <connectionPointOut formalParameter="">
        <relPosition x="142" y="1"/>
      </connectionPointOut>
      <connectionPointOut formalParameter="">
        <relPosition x="282" y="1"/>
      </connectionPointOut>
    </selectionDivergence>
    <step localId="23" name="Step2" initialStep="false" height="23" width
        ="38">
      <position x="199" y="222"/>
      <connectionPointIn>
        <relPosition x="19" y="0"/>
        <connection refLocalId="24">
          <position x="218" y="222"/>
          <position x="218" y="196"/>
        </connection>
      </connectionPointIn>
      <connectionPointOut formalParameter="">
        <relPosition x="19" y="23"/>
      </connectionPointOut>
      <connectionPointOutAction formalParameter="">
        <relPosition x="38" y="11"/>
      </connectionPointOutAction>
    </step>
    <transition localId="24" height="2" width="20">
      <position x="208" y="194"/>
      <connectionPointIn>
        <relPosition x="10" y="0"/>
        <connection refLocalId="22">
          <position x="218" y="194"/>
          <position x="218" y="184"/>
          <position x="218" y="184"/>
          <position x="218" y="176"/>
        </connection>
      </connectionPointIn>
      <connectionPointOut>
        <relPosition x="10" y="2"/>
      </connectionPointOut>
      <condition>
        <inline name="">
```

```
        <ST>
          <xhtml:p><![CDATA[ph >= 7.2]]></xhtml:p>
        </ST>
      </inline>
    </condition>
  </transition>
  <step localId="25" name="Step3" initialStep="false" height="23" width
      ="38">
    <position x="481" y="221"/>
    <connectionPointIn>
      <relPosition x="19" y="0"/>
      <connection refLocalId="26">
        <position x="500" y="221"/>
        <position x="500" y="211"/>
        <position x="500" y="211"/>
        <position x="500" y="196"/>
      </connection>
    </connectionPointIn>
    <connectionPointOut formalParameter="">
      <relPosition x="19" y="23"/>
    </connectionPointOut>
    <connectionPointOutAction formalParameter="">
      <relPosition x="38" y="11"/>
    </connectionPointOutAction>
  </step>
  <transition localId="26" height="2" width="20">
    <position x="490" y="194"/>
    <connectionPointIn>
      <relPosition x="10" y="0"/>
      <connection refLocalId="22">
        <position x="500" y="194"/>
        <position x="500" y="184"/>
        <position x="500" y="184"/>
        <position x="500" y="176"/>
      </connection>
    </connectionPointIn>
    <connectionPointOut>
      <relPosition x="10" y="2"/>
    </connectionPointOut>
    <condition>
      <inline name="">
        <ST>
          <xhtml:p><![CDATA[ph <= 6.8]]></xhtml:p>
        </ST>
      </inline>
    </condition>
  </transition>
```

```
<step localId="27" name="Step4" initialStep="false" height="23" width
    ="38">
  <position x="341" y="394"/>
  <connectionPointIn>
    <relPosition x="19" y="0"/>
    <connection refLocalId="28">
      <position x="360" y="394"/>
      <position x="360" y="370"/>
    </connection>
  </connectionPointIn>
  <connectionPointOutAction formalParameter="">
    <relPosition x="38" y="11"/>
  </connectionPointOutAction>
</step>
<selectionConvergence localId="28" height="1" width="175">
  <position x="272" y="369"/>
  <connectionPointIn>
    <relPosition x="0" y="0"/>
    <connection refLocalId="32">
      <position x="272" y="369"/>
      <position x="272" y="345"/>
    </connection>
  </connectionPointIn>
  <connectionPointIn>
    <relPosition x="88" y="0"/>
    <connection refLocalId="33">
      <position x="360" y="369"/>
      <position x="360" y="303"/>
    </connection>
  </connectionPointIn>
  <connectionPointIn>
    <relPosition x="175" y="0"/>
    <connection refLocalId="1">
      <position x="447" y="369"/>
      <position x="447" y="342"/>
    </connection>
  </connectionPointIn>
  <connectionPointOut>
    <relPosition x="88" y="1"/>
  </connectionPointOut>
</selectionConvergence>
<selectionDivergence localId="29" height="1" width="110">
  <position x="162" y="280"/>
  <connectionPointIn>
    <relPosition x="56" y="0"/>
    <connection refLocalId="23">
      <position x="218" y="280"/>
```

```
        <position x="218" y="270"/>
        <position x="218" y="270"/>
        <position x="218" y="245"/>
      </connection>
    </connectionPointIn>
    <connectionPointOut formalParameter="">
      <relPosition x="0" y="1"/>
    </connectionPointOut>
    <connectionPointOut formalParameter="">
      <relPosition x="110" y="1"/>
    </connectionPointOut>
  </selectionDivergence>
  <selectionDivergence localId="30" height="1" width="106">
    <position x="447" y="268"/>
    <connectionPointIn>
      <relPosition x="53" y="0"/>
      <connection refLocalId="25">
        <position x="500" y="268"/>
        <position x="500" y="250"/>
        <position x="500" y="250"/>
        <position x="500" y="260"/>
      </connection>
    </connectionPointIn>
    <connectionPointOut formalParameter="">
      <relPosition x="0" y="1"/>
    </connectionPointOut>
    <connectionPointOut formalParameter="">
      <relPosition x="106" y="1"/>
    </connectionPointOut>
  </selectionDivergence>
  <selectionConvergence localId="31" height="1" width="40">
    <position x="340" y="104"/>
    <connectionPointIn>
      <relPosition x="0" y="0"/>
      <connection refLocalId="35">
        <position x="340" y="104"/>
        <position x="340" y="86"/>
        <position x="131" y="86"/>
        <position x="131" y="237"/>
        <position x="131" y="237"/>
        <position x="131" y="178"/>
      </connection>
    </connectionPointIn>
    <connectionPointIn>
      <relPosition x="40" y="0"/>
      <connection refLocalId="36">
        <position x="380" y="104"/>
```

```
            <position x="380" y="86"/>
            <position x="683" y="86"/>
            <position x="683" y="302"/>
            <position x="683" y="302"/>
            <position x="683" y="167"/>
          </connection>
        </connectionPointIn>
        <connectionPointOut>
          <relPosition x="20" y="1"/>
        </connectionPointOut>
      </selectionConvergence>
      <transition localId="32" height="13" width="20">
        <position x="262" y="332"/>
        <connectionPointIn>
          <relPosition x="10" y="0"/>
          <connection refLocalId="29">
            <position x="272" y="332"/>
            <position x="272" y="281"/>
          </connection>
        </connectionPointIn>
        <connectionPointOut>
          <relPosition x="10" y="13"/>
        </connectionPointOut>
        <condition>
          <inline name="">
            <ST>
              <xhtml:p><![CDATA[full = TRUE]]></xhtml:p>
            </ST>
          </inline>
        </condition>
      </transition>
      <transition localId="1" height="3" width="20" executionOrderId="0">
        <position x="437" y="339"/>
        <connectionPointIn>
          <relPosition x="10" y="0"/>
          <connection refLocalId="30">
            <position x="447" y="339"/>
            <position x="447" y="269"/>
          </connection>
        </connectionPointIn>
        <connectionPointOut>
          <relPosition x="10" y="3"/>
        </connectionPointOut>
        <condition>
          <inline name="">
            <ST>
              <xhtml:p><![CDATA[full = TRUE]]></xhtml:p>
```

```
      </ST>
    </inline>
  </condition>
</transition>
<transition localId="33" height="2" width="20">
  <position x="350" y="301"/>
  <connectionPointIn>
    <relPosition x="10" y="0"/>
    <connection refLocalId="22">
      <position x="360" y="301"/>
      <position x="360" y="176"/>
    </connection>
  </connectionPointIn>
  <connectionPointOut>
    <relPosition x="10" y="2"/>
  </connectionPointOut>
  <condition>
    <inline name="">
      <ST>
        <xhtml:p><![CDATA[full = TRUE]]></xhtml:p>
      </ST>
    </inline>
  </condition>
</transition>
<actionBlock localId="34" height="30" width="110">
  <position x="398" y="390"/>
  <connectionPointIn>
    <relPosition x="0" y="15"/>
    <connection refLocalId="27">
      <position x="398" y="405"/>
      <position x="379" y="405"/>
    </connection>
  </connectionPointIn>
  <action localId="0" qualifier="P1">
    <relPosition x="0" y="0"/>
    <inline>
      <ST>
        <xhtml:p><![CDATA[pump := FALSE;]]></xhtml:p>
      </ST>
    </inline>
  </action>
</actionBlock>
<transition localId="35" height="2" width="20">
  <position x="121" y="176"/>
  <connectionPointIn>
    <relPosition x="10" y="0"/>
    <connection refLocalId="29">
```

```
          <position x="131" y="176"/>
          <position x="131" y="166"/>
          <position x="131" y="166"/>
          <position x="131" y="301"/>
          <position x="162" y="301"/>
          <position x="162" y="281"/>
        </connection>
      </connectionPointIn>
      <connectionPointOut>
        <relPosition x="10" y="2"/>
      </connectionPointOut>
      <condition>
        <inline name="">
          <ST>
            <xhtml:p><![CDATA[ph <= 7.0]]></xhtml:p>
          </ST>
        </inline>
      </condition>
    </transition>
    <transition localId="36" height="2" width="20">
      <position x="673" y="165"/>
      <connectionPointIn>
        <relPosition x="10" y="0"/>
        <connection refLocalId="30">
          <position x="683" y="165"/>
          <position x="683" y="155"/>
          <position x="683" y="155"/>
          <position x="683" y="308"/>
          <position x="553" y="308"/>
          <position x="553" y="269"/>
        </connection>
      </connectionPointIn>
      <connectionPointOut>
        <relPosition x="10" y="2"/>
      </connectionPointOut>
      <condition>
        <inline name="">
          <ST>
            <xhtml:p><![CDATA[ph >= 7.0]]></xhtml:p>
          </ST>
        </inline>
      </condition>
    </transition>
    <actionBlock localId="37" height="30" width="101">
      <position x="531" y="217"/>
      <connectionPointIn>
        <relPosition x="0" y="15"/>
```

```
            <connection refLocalId="25">
              <position x="531" y="232"/>
              <position x="521" y="232"/>
              <position x="521" y="232"/>
              <position x="519" y="232"/>
            </connection>
          </connectionPointIn>
          <action localId="0" qualifier="P1">
            <relPosition x="0" y="0"/>
            <inline>
              <ST>
                <xhtml:p><![CDATA[vphp :=TRUE;]]></xhtml:p>
              </ST>
            </inline>
          </action>
        </actionBlock>
        <actionBlock localId="38" height="30" width="106">
          <position x="248" y="218"/>
          <connectionPointIn>
            <relPosition x="0" y="15"/>
            <connection refLocalId="23">
              <position x="248" y="233"/>
              <position x="232" y="233"/>
              <position x="232" y="233"/>
              <position x="237" y="233"/>
            </connection>
          </connectionPointIn>
          <action localId="0" qualifier="P1">
            <relPosition x="0" y="0"/>
            <inline>
              <ST>
                <xhtml:p><![CDATA[vphm := TRUE;]]></xhtml:p>
              </ST>
            </inline>
          </action>
        </actionBlock>
      </SFC>
    </body>
  </pou>
```

FCT
Fundação para a Ciência e a Tecnologia

This work is financed by the ERDF - European Regional Development Fund through the COMPETE Programme (operational programme for competitiveness) and by National Funds through the FCT - Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) within project FCOMP-01-0124-FEDER-028923