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A Compositional Semantics for Stochastic Reo Connectors

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In this paper we present a compositional semantics for the channel-based coordination language Reo which enables the analysis of quality of service (QoS) properties of service compositions. For this purpose, we annotate Reo channels with stochastic delay rates and explicitly model data-arrival rates at the boundary of a connector, to capture its interaction with the services that comprise its environment. We propose Stochastic Reo automata as an extension of Reo automata, in order to compositionally derive a QoS-aware semantics for Reo. We further present a translation of Stochastic Reo automata to Continuous-Time Markov Chains (CTMCs). This translation enables us to use third-party CTMC verification tools to do an end-to-end performance analysis of service compositions. As a case study, we are currently applying our method to an industrial application, the ASK system.

1 Introduction

In service-oriented computing (SOC), complex distributed applications are built by composing existing – often third-party – services using additional coordination mechanisms, such as workflow engines, component connectors, or tailor-made glue code. Due to the high degree of heterogeneity and the fact that the owner of the application is not necessarily the owner of its building blocks, issues involving quality of service (QoS) properties become increasingly entangled. Even if the QoS properties of every individual service and connector are known, it is far from trivial to determine and reason about the end-to-end QoS of a composed system in its application context. Yet, the end-to-end QoS of a composed service is often as important as its functional properties in determining its viability in its market.

Reo [2], a channel-based coordination language, supports the composition of services, and typically, its semantics is given by Constraint Automata (CA) [1]. However, CA do not account for the QoS properties and cannot capture the context-dependency [1] of Reo connectors. To capture context-dependency, Reo automata were introduced in [3], but they still do not account for the QoS properties. Quantitative Intentional Automata (QIA) were proposed in [4] to account for the end-to-end QoS properties of a Reo connector, but no formal results are readily available on their compositionality.

As our contribution, we present *Stochastic Reo automata* to overcome the shortcomings of CA and QIA, mentioned above: a compositional semantic model for reasoning about the end-to-end QoS properties, as well as handling the context-dependency of Reo connectors. We show that the compositionality results of Reo automata extend to Stochastic Reo automata. We present a translation of Stochastic Reo automata to Continuous-Time Markov Chains (CTMCs). This allows the use of third-party tools for stochastic analysis. Therefore, this paper shows a compositional approach for constructing Markov Chain (MC) models of complex composite systems, using Stochastic Reo automata as an intermediate model. In other words, Stochastic Reo automata provides a compositional framework wherein the corresponding CTMC model of a connector can be derived. This approach, thus, constitutes a compositional framework for modeling and analysis of the QoS properties of complex systems, where our translation derives a CTMC model for complex systems for subsequent analysis by other tools.

2 Overview of Reo

Reo is a channel-based coordination model wherein so-called *connectors* are used to coordinate, i.e., control the communication among, components or services exogenously (from outside of those components and services). In Reo, complex connectors are compositionally built out of primitive channels. Channels are atomic connectors with exactly two ends, which can be either *source* or *sink* ends. Source ends accept data into, and sink ends dispense data out of their respective channels. Reo allows channels to be undirected, i.e., to have respectively two source or two sink ends.

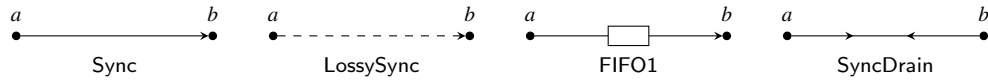


Figure 1: Some basic Reo channels

Figure 1 shows the graphical representations of some basic channel types. The Sync channel is a directed, unbuffered channel that synchronously reads data items from its source end and writes them to its sink end. The LossySync channel behaves similarly, except that it does not block if the party at the sink end is not ready to receive data. Instead, it just loses the data item. FIFO1 is an asynchronous channel with a buffer of size one. The SyncDrain channel differs from the other channels in that it has two source ends (and no sink end). If there is data available at both ends, this channel consumes (and loses) both data items synchronously.

Channels can be joined together using nodes. A node can have one of three types: source, sink or mixed node, depending on whether all ends that coincide on the node are source ends, sink ends or a combination of both. Source and sink nodes, called *boundary nodes*, form the boundary of a connector, allowing interaction with its environment. Source nodes act as synchronous replicators, and sink nodes as mergers. A mixed node combines both behaviors by atomically consuming a data item from one sink end and replicating it to all of its source ends.

An example connector is depicted in Figure 2. It reads a data item from a , buffers it in a FIFO1 and writes it to d . The connector loses data items from a if and only if the FIFO1 buffer is already full. This construct is therefore called (overflow) LossyFIFO1.

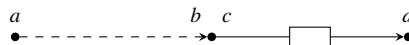


Figure 2: Example connector: LossyFIFO1

2.1 Semantics: Reo automata

In this section, we recall Reo automata, an automata model that provides a compositional operational semantics for Reo connectors. Intuitively, a Reo automaton is a non-deterministic automaton whose transitions are of the form $q \xrightarrow{g|f} q'$, where g is a *guard* (boolean condition) and f a set of nodes that fire synchronously. A transition can be taken only when its guard g is true.

We recall some facts about Boolean algebras. Let $\Sigma = \{\sigma_1, \dots, \sigma_k\}$, $\bar{\sigma}$ be the negation of σ , and \mathcal{B}_Σ be the free Boolean algebra generated by the following grammar:

$$g ::= \sigma \in \Sigma \mid \top \mid \perp \mid g \vee g \mid g \wedge g \mid \bar{g}$$

We refer to the elements of the above grammar as *guards* and in its representation we frequently omit \wedge and write $g_1 g_2$ instead of $g_1 \wedge g_2$. Given two guards $g_1, g_2 \in \mathcal{B}_\Sigma$, we define a (natural) order \leq as $g_1 \leq g_2 \iff g_1 \wedge g_2 = g_1$. The intended interpretation of \leq is logical implication: g_1 implies g_2 . An *atom* of \mathcal{B}_Σ is a guard $a_1 \dots a_k$ such that $a_i \in \Sigma \cup \bar{\Sigma}$ with $\bar{\Sigma} = \{\bar{\sigma}_i \mid \sigma_i \in \Sigma\}$, $1 \leq i \leq k$. We can think of an atom as a truth assignment. We denote atoms by Greek letters α, β, \dots and the set of all atoms of \mathcal{B}_Σ by \mathbf{At}_Σ . Given $S \subseteq \Sigma$, we define $\widehat{S} \in \mathcal{B}_\Sigma$ as the conjunction of all elements of S . For instance, for $S = \{a, b, c\}$ we have $\widehat{S} = abc$.

Definition 1 [3] A Reo automaton is a triple (Σ, Q, δ) where Σ is the set of nodes, Q is the set of states,

$\delta \subseteq Q \times \mathcal{B}_\Sigma \times 2^\Sigma \times Q$ is the transition relation such that for each $q \xrightarrow{g|f} q' \in \delta$:

- (i) $g \leq \widehat{f}$ (reactivity)
- (ii) $\forall g \leq g' \leq \widehat{f} \cdot \forall \alpha \leq g' \cdot \exists q \xrightarrow{g'|f} q' \in \delta \cdot \alpha \leq g''$ (uniformity)

In Reo automata, for simplicity we abstract data constraints [1] and assume they are *true*. We use arrows $q \xrightarrow{g|f} q'$ for $\langle q, g, f, q' \rangle \in \delta$. If there is more than one transition from state q to q' we often just draw one arrow and separate their labels by commas. In Figure 3 we depict the Reo automata for the basic channel types listed in Figure 1.

Intuitively, every transition $q \xrightarrow{g|f} q'$ in an automaton corresponding to a Reo connector represents that, if the connector is in state q and the boundary requests present at the moment, encoded by an atom α , are such that $\alpha \leq g$, then the nodes f fires and the connector evolves to state q' . Each transition labeled by $g|f$ satisfies two criteria: (i) *reactivity*—data flows only on nodes where a request is pending, capturing Reo’s interaction model; and (ii) *uniformity*—which captures two properties, firstly, that the request set corresponding precisely to the firing set is sufficient to cause firing, and secondly, that removing additional unfired requests from a transition will not affect the (firing) behavior of the connector [3].

2.1.1 Composing Reo connectors

We now model at the automata level the composition of Reo connectors. We define two operations: product, which puts two connectors in parallel, and synchronization, which models the plugging of two nodes. Thus, the product and synchronization operations can be used to obtain the automaton of a Reo connector by composing the automata of its primitive connectors. Later in this section we formally show the compositionality of the operations.

We first define the product operation for Reo automata. This definition differs from the classical definition of (synchronous) product for automata: our automata have disjoint alphabets and they can either take steps together or independently. In the latter case the composite transition in the product

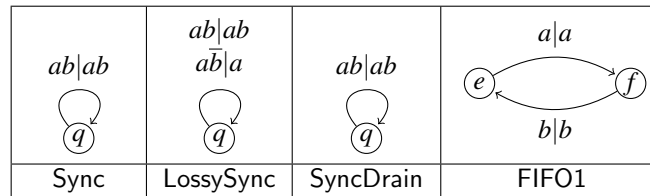


Figure 3: Automata for basic Reo channels

automaton explicitly encodes that one of the two automata cannot perform a step in the current state, using the following notion:

Definition 2 [3] Given a Reo automaton $\mathcal{A} = (\Sigma, Q, \delta)$ and $q \in Q$ we define

$$q^\# = \neg \bigvee \{ g \mid q \xrightarrow{g|f} q' \in \delta \}.$$

This captures precisely that \mathcal{A} cannot fire in state q .

Definition 3 [3] Given two Reo automata $\mathcal{A}_1 = (\Sigma_1, Q_1, \delta_1)$ and $\mathcal{A}_2 = (\Sigma_2, Q_2, \delta_2)$ such that $\Sigma_1 \cap \Sigma_2 = \emptyset$, we define the product of \mathcal{A}_1 and \mathcal{A}_2 as $\mathcal{A}_1 \times \mathcal{A}_2 = (\Sigma_1 \cup \Sigma_2, Q_1 \times Q_2, \delta)$ where δ consists of:

$$\begin{aligned} & \{(q, p) \xrightarrow{gg'|ff'} (q', p') \mid q \xrightarrow{g|f} q' \in \delta_1 \wedge p \xrightarrow{g'|f'} p' \in \delta_2\} \\ & \cup \{(q, p) \xrightarrow{gp^\#|f} (q', p) \mid q \xrightarrow{g|f} q' \in \delta_1 \wedge p \in Q_2\} \\ & \cup \{(q, p) \xrightarrow{gq^\#|f} (q, p') \mid p \xrightarrow{g'|f'} p' \in \delta_2 \wedge q \in Q_1\} \end{aligned}$$

Here and throughout, we use ff' as a shorthand for $f \cup f'$. The first term in the union, above, applies when both automata fire in parallel. The other terms apply when one automaton fires and the other is unable to (given by $p^\#$ and $q^\#$, respectively). Note that the product operation is closed for Reo automata, since it preserves reactivity and uniformity [3]. Figure 4 shows an example of the product of two automata.

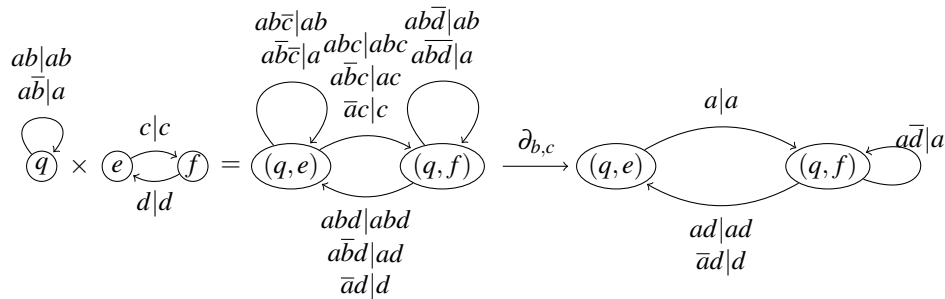


Figure 4: Product of LossySync and FIFO1 and its synchronization of nodes b and c

We now define a synchronization operation that corresponds to joining two nodes in a Reo connector. In order for this operation to be well-defined we need that every guard in a transition label in the automata is a conjunction of literals. Note that in the automata presented in Figure 3 for basic Reo channels this is already the case, and moreover, it is always possible to transform any guard g into this form, by taking its disjunctive normal form (DNF) $g_1 \vee \dots \vee g_k$ and splitting the transition $g|f$ into the several $g_i|f$, for $i = 1, \dots, k$. Given a transition relation δ we call $norm(\delta)$ the normalized transition relation obtained from δ by putting all its guards in DNF and splitting the transitions as explained above.

When synchronizing two nodes a and b (which are then made internal), in the resulting automaton, only the transitions where either both a and b or neither a nor b fire are kept — that is, a and b synchronize. In order to propagate context information (requests), we require that every guard contains either a or b , expressed by the condition $g \not\leq \overline{ab}$ below, which more or less corresponds to an internal node acting like a *self-contained pumping station* [2], meaning that an internal node cannot store data or actively block behavior.

Definition 4 [3] Given a Reo automaton $\mathcal{A} = (\Sigma, Q, \delta)$, we define the synchronization for $a, b \in \Sigma$ as $\partial_{a,b}\mathcal{A} = (\Sigma, Q, \delta')$ where

$$\delta' = \{q \xrightarrow{g \setminus_{ab} | f \setminus_{\{a,b\}}} q' \mid q \xrightarrow{g|f} q' \in \text{norm}(\delta) \text{ s.t. } g \not\leq \bar{a}\bar{b} \text{ and } a \in f \Leftrightarrow b \in f\}$$

Here and throughout, $g \setminus_{ab}$ is the guard obtained from g by deleting all occurrences of a and b . It is worth noting that synchronization preserves reactivity and uniformity.

Figure 4 depicts the product of LossySync and FIFO1, together with the result of synchronizing nodes b and c . This synchronized result provides the semantics for the LossyFIFO1 example in Figure 2.

2.1.2 Compositionality

Given two Reo automata \mathcal{A}_1 and \mathcal{A}_2 over the disjoint alphabets Σ_1 and Σ_2 , $\{a_1, \dots, a_k\} \subseteq \Sigma_1$ and $\{b_1, \dots, b_k\} \subseteq \Sigma_2$ we construct $\partial_{a_1, b_1} \partial_{a_2, b_2} \dots \partial_{a_k, b_k} (\mathcal{A}_1 \times \mathcal{A}_2)$ as the automaton corresponding to a connector where node a_i of the first connector is connected to node b_i of the second connector, for all $i \in \{1, \dots, k\}$. Note that the ‘plugging’ order does not matter because ∂ is commutative and it interacts well with product. These properties are captured in the following lemma.

Lemma 1 [3] *For the Reo automata $\mathcal{A}_1 = (\Sigma_1, Q_1, \delta_1)$ and $\mathcal{A}_2 = (\Sigma_2, Q_2, \delta_2)$:*

1. $\partial_{a,b} \partial_{c,d} \mathcal{A}_1 = \partial_{c,d} \partial_{a,b} \mathcal{A}_1$, if $a, b, c, d \in \Sigma_1$.
2. $(\partial_{a,b} \mathcal{A}_1) \times \mathcal{A}_2 \sim \partial_{a,b} (\mathcal{A}_1 \times \mathcal{A}_2)$, if $a, b \notin \Sigma_2$

The notion of equivalence \sim used above is bisimulation, defined as follows.

Definition 5 [3] *Given the Reo automata $\mathcal{A}_1 = (\Sigma, Q_1, \delta_1)$ and $\mathcal{A}_2 = (\Sigma, Q_2, \delta_2)$, we call $R \subseteq Q_1 \times Q_2$ a bisimulation iff for all $(q_1, q_2) \in R$:*

If $q_1 \xrightarrow{g|f} q'_1 \in \delta_1$ and $\alpha \in \mathbf{At}_\Sigma$, $\alpha \leq g$, then there exists a transition $q_2 \xrightarrow{g'|f} q'_2 \in \delta_2$ such that $\alpha \leq g'$ and $(q'_1, q'_2) \in R$ and vice-versa.

We say that two states $q_1 \in Q_1$ and $q_2 \in Q_2$ are bisimilar if there exists a bisimulation relation containing the pair (q_1, q_2) and we write $q_1 \sim q_2$. Two automata \mathcal{A}_1 and \mathcal{A}_2 are bisimilar, written $\mathcal{A}_1 \sim \mathcal{A}_2$, if there exists a bisimulation relation such that every state of one automaton is related to some state of the other automaton.

3 Stochastic Reo

Stochastic Reo is an extension of Reo where channel ends and channels are annotated with stochastic values for *data arrival rates* at channel ends and *processing delay rates* at channels. Such rates are non-negative real values and describe how the probability that an event occurs varies with time. Figure 5 shows the stochastic versions of the primitive Reo channels in Figure 1. Here and throughout, for simplicity, we omit the node names, since they can be inferred from the names of their respective arrival rates: for instance, γa is the arrival rate of node a .

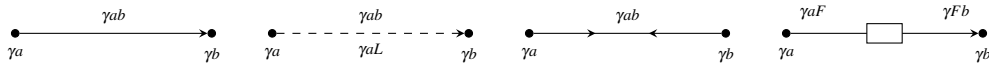


Figure 5: Basic Stochastic Reo channels

A processing delay rate represents how long it takes for a channel to perform a certain activity, such as data-flow. For instance, a LossySync has two associated rates γab and γaL for, respectively, successful

data-flow from node a to node b , and losing the data item from node a . In a FIFO1 γaF represents the delay for data-flow from its source a into the buffer, and γFb for sending the data from the buffer to the sink b .

Arrival rates describe the time between consecutive arrivals of I/O requests at the source and sink nodes of Reo connectors. For instance, γa and γb in Figure 5 are the associated arrival rates of write/take requests at the nodes a and b .

Since arrival rates on nodes model their interaction with the environment only, mixed nodes have no associated arrival rates. This is justified by the fact that a mixed node delivers data items instantaneously to the source end(s) of its connected channel(s). Hence, when joining a source with a sink node into a mixed node, their arrival rates are discarded¹.

A stochastic version of the LossyFIFO1 is depicted in Figure 6, including its arrival and processing delay rates.

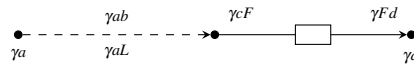


Figure 6: Stochastic LossyFIFO1

3.1 Semantics: Stochastic Reo automata

In this section, we provide a compositional semantics for Stochastic Reo connectors, as an extension of Reo automata with functions that assign stochastic values for data-flows and I/O request arrivals.

Definition 6 A Stochastic Reo automaton is a triple $(\mathcal{A}, \mathbf{r}, \mathbf{t})$ where $\mathcal{A} = (\Sigma, Q, \delta_{\mathcal{A}})$ is a Reo automaton and

- $\mathbf{r} : \Sigma \rightarrow \mathbb{R}^+$ is a function that associates with each node its arrival rate.
- $\mathbf{t} : \delta_{\mathcal{A}} \rightarrow 2^{\Theta}$ is a function that associates with a transition a subset of $\Theta \subseteq 2^{\Sigma} \times 2^{\Sigma} \times \mathbb{R}^+$ such that each $(I, O, r) \in \Theta$ corresponds to a data-flow where I is a set of input and/or mixed nodes; O is a set of output and/or mixed nodes and r is a processing delay rate for the data-flow.

LossySync and FIFO1 constituting LossyFIFO1 in Figure 6 are defined with the functions \mathbf{r} and \mathbf{t} in Table 1. Note that the function \mathbf{t} is applied to and put along with its associated transition, and function \mathbf{r} is shown by a table.



$ab ab, \{(\{a\}, \{b\}, \gamma ab)\}$ $ab a, \{(\{a\}, \emptyset, \gamma aL)\}$ 	<table border="1" style="border-collapse: collapse;"> <tr><td></td><td>r</td></tr> <tr><td>a</td><td>γa</td></tr> <tr><td>b</td><td>γb</td></tr> </table>		r	a	γa	b	γb	$c c, \{(\{c\}, \emptyset, \gamma cF)\}$  $d d, \{(\emptyset, \{d\}, \gamma Fd)\}$	<table border="1" style="border-collapse: collapse;"> <tr><td></td><td>r</td></tr> <tr><td>c</td><td>γc</td></tr> <tr><td>d</td><td>γd</td></tr> </table>		r	c	γc	d	γd
	r														
a	γa														
b	γb														
	r														
c	γc														
d	γd														

Table 1: Stochastic Reo automaton for LossySync and FIFO1

¹For simplicity, we assume ideal nodes whose activity incurs no delay. Any real implementation of a node, of course, induces some processing delay rate. A real node can be modeled as a composition of an ideal node with a Sync channel that manifests the processing delay rate. Thus, we can associate delay distributions with Stochastic Reo nodes and automatically translate them into such “Sync plus ideal node” constructs .

An element of $\theta \in \Theta$ is accessed by projection functions $i : \Theta \rightarrow 2^\Sigma$, $o : \Theta \rightarrow 2^\Sigma$ and $v : \Theta \rightarrow \mathbb{R}^+$; $i(\theta)$ and $o(\theta)$ return, respectively, relevant input and output nodes of a data-flow, and $v(\theta)$ returns the delay rate of a data-flow through nodes in $i(\theta)$ and $o(\theta)$.

Definition 7 Given two Stochastic Reo automata $(\mathcal{A}_1, \mathbf{r}_1, \mathbf{t}_1)$ and $(\mathcal{A}_2, \mathbf{r}_2, \mathbf{t}_2)$, their product is defined as $(\mathcal{A}_1, \mathbf{r}_1, \mathbf{t}_1) \times (\mathcal{A}_2, \mathbf{r}_2, \mathbf{t}_2) = (\mathcal{A}_1 \times \mathcal{A}_2, \mathbf{r}_1 \cup \mathbf{r}_2, \mathbf{t})$ where

$$\begin{aligned} \mathbf{t}((q, p) \xrightarrow{g|f|f'} (q', p')) &= \mathbf{t}_1(q \xrightarrow{g|f} q') \cup \mathbf{t}_2(p \xrightarrow{g'|f'} p') \\ \mathbf{t}((q, p) \xrightarrow{g|f} (q', p)) &= \mathbf{t}_1(q \xrightarrow{g|f} q') \\ \mathbf{t}((q, p) \xrightarrow{g'|f'} (q, p')) &= \mathbf{t}_2(p \xrightarrow{g'|f'} p') \end{aligned}$$

Note that we use \times to denote both the product of Reo automata and the product of Stochastic Reo automata.

The set of 3-tuples that \mathbf{t} associates with a transition m represents the composition of the delay rates involved in all data-flows synchronized by the transition m . In order to keep Stochastic Reo automata generally useful and compositional, and their product commutative, we avoid fixing the precise formal meaning of distribution rates of synchronized transitions composed in a product; instead, we present the “delay rate” of their composite transition in the product automaton as the union of the delay rates of the synchronizing transitions of the two automata. How exactly these rates combine to yield the composite rate of the transition depends on different properties of the distributions and their time ranges. For example, in the continuous-time case, no two events can occur at the same time; whereas the exponential distributions are not closed under taking maximum. In Section 4 we show how to translate a Stochastic Reo automaton to a CTMC by the union of rates of the exponential distribution in the continuous-time case.

Definition 8 For a Stochastic Reo automaton $(\mathcal{A}, \mathbf{r}, \mathbf{t})$, the synchronization operation on nodes a and b is defined as $\partial_{a,b}(\mathcal{A}, \mathbf{r}, \mathbf{t}) = (\partial_{a,b}\mathcal{A}, \mathbf{r}', \mathbf{t}')$ where

- \mathbf{r}' is \mathbf{r} restricted to the domain $\Sigma \setminus \{a, b\}$.
- \mathbf{t}' is defined as:

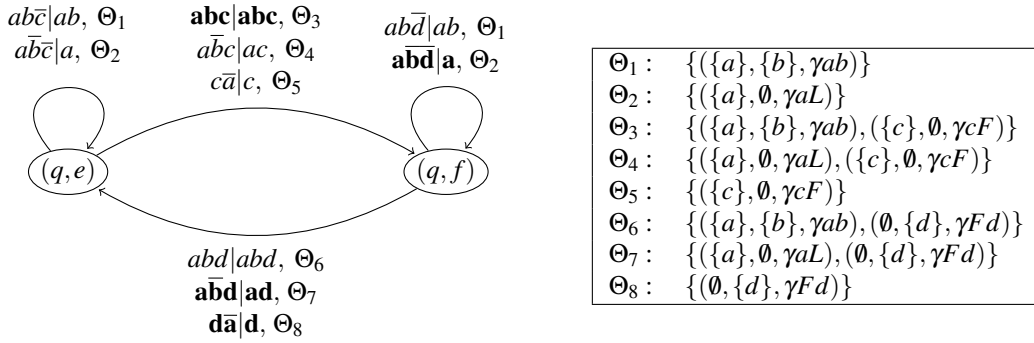
$$\mathbf{t}'(q \xrightarrow{g|ab|f|f'} \{a,b\} q') = \{ (A', B', r) \mid (A, B, r) \in \mathbf{t}(q \xrightarrow{g|f} q'), \\ A' = \text{sync}(A, \{a, b\}) \wedge B' = \text{sync}(B, \{a, b\}) \}$$

- $\text{sync} : 2^\Sigma \times 2^\Sigma \rightarrow 2^\Sigma$ gathers nodes joined by synchronization, and is defined as:

$$\text{sync}(A, B) = \begin{cases} A \cup B & \text{if } A \cap B \neq \emptyset \\ A & \text{otherwise} \end{cases}$$

Note that we use the symbol $\partial_{a,b}$ to denote both the synchronization of Reo automata and the synchronization of Stochastic Reo automata.

We now revisit the LossyFIFO1 example. Its semantics is given by the triple $(\mathcal{A}_{\text{LossyFIFO1}}, \mathbf{r}, \mathbf{t})$, where $\mathcal{A}_{\text{LossyFIFO1}}$ is the automaton depicted in Figure 4 and \mathbf{r} is defined as $\mathbf{r} = \{a \mapsto \gamma a, d \mapsto \gamma d\}$. For \mathbf{t} , we first compute $\mathbf{t}_{\text{LossySync} \times \text{FIFO1}}$:



Above, the labels that correspond to the transitions that will be kept after synchronization appear in **bold**. Thus, the result of joining nodes by synchronization, is shown in Figure 7 as:

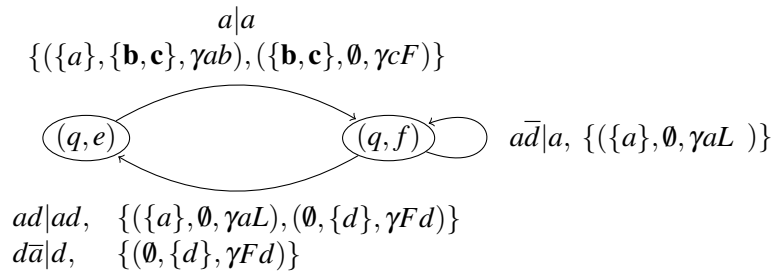


Figure 7: Stochastic Reo automaton for LossyFIFO1

Note that the port names that appear in **bold** represent the synchronization of nodes b and c .

In this way, we can carry in the semantic model of Reo circuits, given as Reo automata, stochastic information, i.e., arrival rates and processing delay rates that pertain to its QoS.

Definition 6 shows that our extension of Reo automata deals with such stochastic information separately, apart from the underlying Reo automaton. Thus, our extended model retains the properties of Reo automata, i.e., the compositionality result presented in Section 2.1.2 can be extended to Stochastic Reo automata:

Lemma 2 For two disjoint Stochastic Reo automata $(\mathcal{A}_1, \mathbf{r}_1, \mathbf{t}_1)$ and $(\mathcal{A}_2, \mathbf{r}_2, \mathbf{t}_2)$,

1. $\partial_{a,b}\partial_{c,d}(\mathcal{A}_1, \mathbf{r}_1, \mathbf{t}_1) = \partial_{c,d}\partial_{a,b}(\mathcal{A}_1, \mathbf{r}_1, \mathbf{t}_1)$
2. $(\partial_{a,b}(\mathcal{A}_1, \mathbf{r}_1, \mathbf{t}_1)) \times (\mathcal{A}_2, \mathbf{r}_2, \mathbf{t}_2) \sim \partial_{a,b}((\mathcal{A}_1, \mathbf{r}_1, \mathbf{t}_1) \times (\mathcal{A}_2, \mathbf{r}_2, \mathbf{t}_2))$

Here $(\mathcal{A}_1, \mathbf{r}_1, \mathbf{t}_1) \sim (\mathcal{A}_2, \mathbf{r}_2, \mathbf{t}_2)$ if and only if $\mathcal{A}_1 \sim \mathcal{A}_2$, $\mathbf{r}_1 = \mathbf{r}_2$ and $\mathbf{t}_1 = \mathbf{t}_2$.

4 Translation to CTMC

In this section, we show how to translate a Stochastic Reo automaton into a homogeneous CTMC model. A homogeneous CTMC is a stochastic process with 1) homogeneity, 2) memoryless/Markov property, and 3) discrete state space in the continuous-time domain [19]. These properties yield efficient methodologies for numerical analysis.

In the continuous-time domain, the exponential distribution is the only one that satisfies the memoryless property. Therefore, for the translation, we assume that the rates of data-arrivals and data-flows are exponentially distributed.

A CTMC model derived from a Stochastic Reo automaton $(\mathcal{A}, \mathbf{r}, \mathbf{t})$ with $\mathcal{A} = (\Sigma, Q, \delta_{\mathcal{A}})$ is a pair (S, δ) where $S = S_A \cup S_M$ is the set of states. S_A represents the configurations of the system derived from its Reo automaton and the pending status of I/O requests; S_M is the set of states that result from the micro-step division of synchronous actions (see below). $\delta = \delta_{Arr} \cup \delta_{Proc} \subseteq S \times \mathbb{R}^+ \times S$, explained below, is the set of transitions, each labeled with a stochastic value specifying the arrival or the processing delay rate of the transition. δ_{Arr} and δ_{Proc} are defined in Section 4.3.

A state in S models a configuration of the connector, including the presence of the I/O requests pending on its boundary nodes, if any. Data-arrivals change system configuration only by changing the pending status of their respective boundary nodes. Data-flows corresponding to a transition of a Reo automaton change the system configuration, and release the pending I/O requests on its involved boundary nodes.

In a CTMC model, the probability that two events (e.g., the arrival of an I/O request, the transfer of a data item, a processing step, etc.) happen at the same time is *zero*: only a single event occurs at a time. In compliance with this requirement, for a Stochastic Reo automaton $(\mathcal{A}, \mathbf{r}, \mathbf{t})$ with $\mathcal{A} = (\Sigma, Q, \delta_{\mathcal{A}})$ and a set of boundary nodes $\Sigma' \subseteq \Sigma$, the set S_A and the preliminary set of data-arrival transitions of the CTMC derived for $(\mathcal{A}, \mathbf{r}, \mathbf{t})$ are defined as:

$$\begin{aligned} S_A &= \{(q, R) \mid q \in Q, R \subseteq \Sigma'\} \\ \delta'_{Arr} &= \{(q, R) \xrightarrow{\mathbf{r}(c)} (q, R \cup \{c\}) \mid (q, R), (q, R \cup \{c\}) \in S_A, c \notin R\} \end{aligned}$$

The set δ'_{Arr} is used in Section 4.3 to define δ_{Arr} .

4.1 Micro-step transitions

The CTMC transitions associated with data-flows are more complicated since groups of synchronized data-flows are modeled as a single transition in a Reo automaton. Therefore, we need to divide such synchronized data-flows into so-called micro-step transitions, respecting the connection information, i.e., the topology of a Reo connector.

The connection information can be recovered from the 3-tuples associated with each transition in a Reo automaton since the first and the second elements of a 3-tuple describe the input and the output nodes, respectively, involved in the data-flow of its transition, and the data-flow in the transition occurs from its input to its output nodes.

For example, the transition $(q, e) \xrightarrow{a|a} (q, f)$ in the Reo automaton of the LossyFIFO1 example in Figure 7 has a set of the 3-tuples $\{(\{a\}, \{b, c\}, \gamma ab), (\{b, c\}, \emptyset, \gamma cF)\}$. The connection information inferred from this set states that data-flow occurs from a to the buffer through b and c . The transition is thus divided into two consecutive micro-step transitions $(\{a\}, \{b, c\}, \gamma ab)$ and $(\{b, c\}, \emptyset, \gamma cF)$.

Such data-flow information of each transition in a Reo automaton is formalized by a *delay-sequence* defined by the following grammar:

$$\Lambda \ni \lambda ::= \varepsilon \mid \theta \mid \lambda \mid \lambda \mid \lambda; \lambda$$

where ε is the empty sequence and θ is a 3-tuple (I, O, r) for a primitive Reo channel. $\lambda \mid \lambda$ denotes parallel composition, and $\lambda; \lambda$ denotes sequential composition. The empty sequence ε is an identity element for $;$ and \mid is commutative, associative, and idempotent, $;$ is associative and distributes over \mid .

4.2 Extracting delay-sequences

The delay-sequence corresponding to a set of 3-tuples associated with a transition in a Stochastic Reo automaton is obtained by Algorithm 4.2.1. Note that if the parameter of the function **Ext** is a singleton,

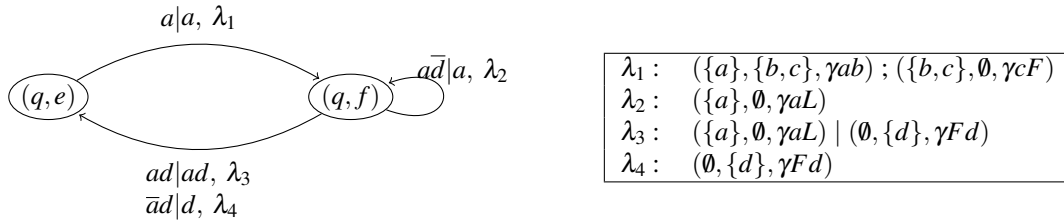
then $\mathbf{Ext}(\{\theta\}) = \theta$ since $i(\theta) \cap o(\theta) = \emptyset$.

Algorithm 4.2.1 Extraction of a delay-sequence

Ext(Θ) where $\Theta = \mathbf{t}(p \xrightarrow{g|f} q)$
 $S = \varepsilon$, $toGo = \Theta$, $Init := \{\theta \in \Theta \mid i(\theta) \cap o(\theta') = \emptyset \text{ for all } \theta' \in \Theta\}$
for $\theta \in Init$ **do**
 $\lambda_\theta := \theta$, $Pre := \{\theta\}$, $toGo := toGo \setminus Pre$
 $Post = \{\theta \in toGo \mid \exists \theta' \in Pre \text{ s.t. } o(\theta') \cap i(\theta) \neq \emptyset\}$
while $Post \neq \emptyset$ **do**
 $\lambda' := (\theta_1 | \dots | \theta_k)$ where $Post = \{\theta_1, \dots, \theta_k\}$
 $\lambda_\theta := \lambda_\theta; \lambda'$, $Pre := Post$, $toGo := toGo \setminus Pre$
 $Post := \{\theta \in toGo \mid \exists \theta' \in Pre \text{ s.t. } o(\theta') \cap i(\theta) \neq \emptyset\}$
end while
 $S := S | \lambda_\theta$
end for
return S

Intuitively, the **Ext** function delineates the set of activities that – at the level of a Stochastic Reo automaton – must happen synchronously/atomically, into corresponding delay-sequences. If a certain data-flow associated with a 3-tuple θ_1 explicitly precedes another one θ_2 , then θ_1 is sequenced before θ_2 , i.e., encoded as $\theta_1; \theta_2$. Otherwise, they can occur in any order, encoded as $\theta_1 | \theta_2$.

Applying Algorithm 4.2.1 to the LossyFIFO1 example yields the following result:



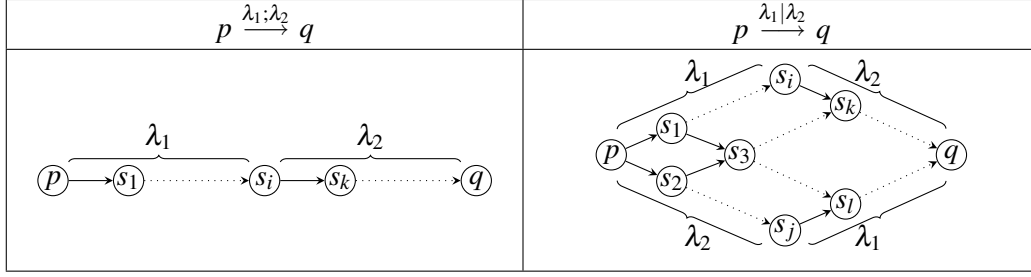
The parameter Θ of Algorithm 4.2.1 is a finite set of 3-tuples, and $Init$, $Post$ and $toGo$, subsets of Θ , are also finite. Moreover, $Post$ becomes eventually \emptyset since $toGo$ decreases during the procedure. Thus, we can conclude that Algorithm 4.2.1 always terminates.

4.3 Deriving the CTMC

We now show how to derive the transitions in the CTMC model from the transitions in a stochastic Reo automaton. We do this in two steps:

1. For each transition $p \xrightarrow{g|f} q \in \delta_A$, we derive transitions $(p, R) \xrightarrow{\lambda} (q, R \setminus f)$ for every set of pending requests R that suffices to activate the guard g ($\widehat{R} \leq g \setminus \widehat{\Sigma}$), where λ is the delay-sequence associated with the set of 3-tuples $\mathbf{t}(p \xrightarrow{g|f} q)$. This set of derived transitions is defined below as δ_{Macro} .
2. We divide a transition in δ_{Macro} labeled by λ into a combination of micro-step transitions, each of which corresponds to a single event.

The following figure briefly illustrates the procedure mentioned above:



A sequential delay-sequence $\lambda_1; \lambda_2$ allows for the events corresponding to λ_1 to occur before the ones corresponding to λ_2 . For a parallel delay-sequence $\lambda_1 | \lambda_2$, events corresponding to λ_1 and λ_2 occur in an interleaving way, while they preserve their respective order of occurrence in λ_1 and λ_2 . All indexed states s_n are included in S_M which consists of the states derived from the division of synchronized data-flows into micro-step transitions.

Given a Stochastic Reo automaton $(\mathcal{A}, \mathbf{r}, \mathbf{t})$ with $\mathcal{A} = (\Sigma, Q, \delta_{\mathcal{A}})$ and a set of boundary nodes Σ' , a macro-step transition relation for the synchronized data-flows is defined as:

$$\delta_{Macro} = \{(p, R) \xrightarrow{\lambda} (q, R \setminus f) \mid p \xrightarrow{g|f} q \in \delta_{\mathcal{A}}, R \subseteq \Sigma', \widehat{R} \leq g \setminus \widehat{\Sigma}, \lambda = \mathbf{Ext}(\mathbf{t}(p \xrightarrow{g|f} q))\}$$

We explicate a macro-step transition with a number of micro-step transitions, each of which corresponds to a single data-flow. This refinement yields auxiliary states between the source and the target states of the macro-step transition. Let (p, R) be a source state for a data-flow corresponding to a 3-tuple θ . Then the generated auxiliary states are defined as $(p_{\theta}, R \setminus nodes(\theta))$ where p_{θ} is just a label denoting that data-flows corresponding to θ have occurred, and the function $nodes : \Lambda \rightarrow 2^{\Sigma}$ is defined as:

$$nodes(\lambda) = \begin{cases} i(\theta) \cup o(\theta) & \text{if } \lambda = \theta \\ nodes(\lambda_1) \cup nodes(\lambda_2) & \text{if } \lambda = \lambda_1; \lambda_2 \vee \lambda = \lambda_1 | \lambda_2 \end{cases}$$

The set of such auxiliary states is obtained as $S_M = states((p, R) \xrightarrow{\lambda} (q, R'))$ where

$$states((p, R) \xrightarrow{\lambda} (q, R')) = \begin{cases} \{(p, R), (q, R')\} & \text{if } \lambda = \theta \\ \bigcup states(m) \forall m \in div((p, R) \xrightarrow{\lambda} (q, R')) & \text{otherwise} \end{cases}$$

The function $div : \delta_{Macro} \rightarrow 2^{\delta_{Macro}}$ is defined as:

$$div((p, R) \xrightarrow{\lambda} (q, R')) = \begin{cases} \{(p, R) \xrightarrow{\theta} (q, R')\} & \text{if } \lambda = \theta \wedge \nexists (p, R) \xrightarrow{\theta} (p', R') \in \delta_{Macro} \\ div((p, R) \xrightarrow{\lambda_1} (p_{\lambda_1}, R'')) \cup div((p_{\lambda_1}, R'') \xrightarrow{\lambda_2} (q, R')) & \text{if } \lambda = \lambda_1; \lambda_2 \text{ where } R'' = R \setminus nodes(\lambda_1) \\ \{m_1 \bowtie m_2 \mid m_i \in div((p, R) \xrightarrow{\lambda_i} (p_{\lambda_i}, R'')), i \in \{1, 2\}\} & \text{if } \lambda = \lambda_1 | \lambda_2 \text{ where } R'' = R \setminus nodes(\lambda_i) \\ \emptyset & \text{otherwise} \end{cases}$$

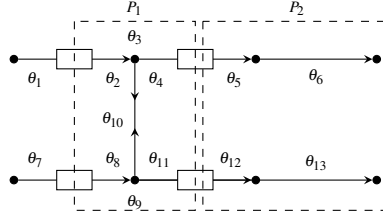
where the function \bowtie computes all interleaving compositions of the two transitions as: for every $(p, R_1) \in states(s_2 \xrightarrow{\theta_2} s'_2)$ and for every $(p, R_2) \in states(s_1 \xrightarrow{\theta_1} s'_1)$

$$\frac{s_1 \xrightarrow{\theta_1} s'_1 \quad \bowtie \quad s_2 \xrightarrow{\theta_2} s'_2}{(p, R_1) \xrightarrow{\theta_1} (p_{\theta_1}, R_1 \setminus nodes(\theta_1))} \qquad \frac{s_1 \xrightarrow{\theta_1} s'_1 \quad \bowtie \quad s_2 \xrightarrow{\theta_2} s'_2}{(p, R_2) \xrightarrow{\theta_2} (p_{\theta_2}, R_2 \setminus nodes(\theta_2))}$$

The following example shows the application of the function *div* to a non-trivial delay-sequence, which contains a combination of sequential and parallel compositions.

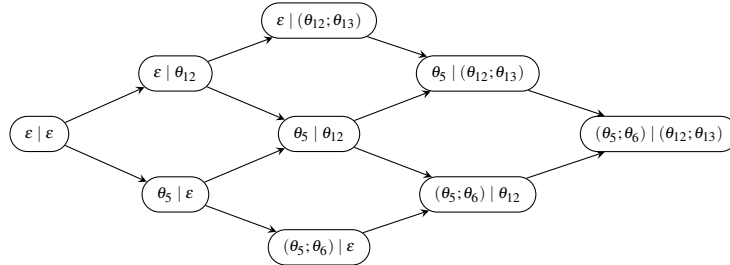
Example 1 Consider the stochastic Reo connector below. Every indexed θ is a rate for its respective processing activity, e.g., θ_2 is the rate at which the top-left FIFO1 dispenses data through its sink end; θ_3 is the rate at which the node replicates its coming data, etc. P_1 and P_2 show up in δ_{Macro} , derived from the Stochastic Reo automaton of this circuit, by two transitions with the delay-sequences of λ_1 and λ_2 where:

- from P_1 : $\lambda_1 = ((\theta_2; \theta_3) | (\theta_8; \theta_9)) ; (\theta_4 | \theta_{10} | \theta_{11})$
- from P_2 : $\lambda_2 = (\theta_5; \theta_6) | (\theta_{12}; \theta_{13})$



To derive a CTMC, λ_1 and λ_2 must be divided into micro-step transitions. We exemplify a few of these divisions. For λ_1 , the division of $(\theta_4 | \theta_{10} | \theta_{11})$ is trivial since it contains only simple parallel composition. This division result is then appended to the division result of $(\theta_2; \theta_3) | (\theta_8; \theta_9)$, which has the same structure as that of λ_2 . Thus, we show below the division result of λ_2 only.

In the following CTMC fragment, to depict which events have occurred up to a current state, the name of each state shows the delays of all the events that have occurred up to the current state. The delay for a newly occurring event is appended to the existing state name.



This example shows that when a delay-sequence λ is generated by parallel composition, the events in one of the sub-delay-sequences of λ occur independently of the events in other sub-delay-sequences. Still they keep their occurrence order in the sub-delay-sequence that they belong to. ■

The division into micro-step transitions ensures that each transition has a single 3-tuple in its label. Thus, the micro-step transitions can be extracted as:

$$\delta_{Proc} = \{(p, R) \xrightarrow{\nu(\theta)} (p', R') \mid (p, R) \xrightarrow{\theta} (p', R') \in \text{div}(t) \text{ for all } t \in \delta_{Macro}\}$$

Synchronized data-flows in a Reo automaton are considered atomic, hence other events cannot interfere with them. However, splitting these data-flows allows non-interfering events to interleave with their micro-steps, disregarding the strict sense of their atomicity. For example, a certain boundary node unrelated to a group of synchronized data-flows can accept a data item between any two micro-steps. Since we want to allow such interleaving, we must explicitly add such data-arrivals. For a Stochastic Reo automaton $(\mathcal{A}, \mathbf{r}, \mathbf{t})$ with $\mathcal{A} = (\Sigma, Q, \delta_{\mathcal{A}})$ and a set of micro-step states S_M , its full set of data-arrival transitions, including its data-arrivals, is defined as:

$$\delta_{Arr} = \delta'_{Arr} \cup \{(p, R) \xrightarrow{r^{(d)}} (p, R \cup \{d\}) \mid (p, R), (p, R \cup \{d\}) \in S_M, d \in \Sigma, d \notin R\}$$

The derived CTMC model can be used for stochastic analysis. For instance, Figure 9 is obtained from PRISM [17] using the CTMC model (see Figure 8) derived from the Stochastic Reo circuit of the LossyFIFO1 example in Figure 6. Figure 9 shows how the probability of data loss varies as the arrival rate at node a increases.

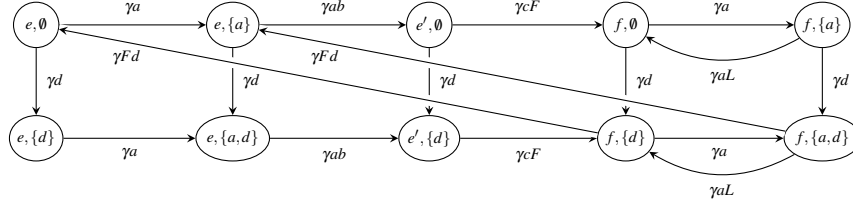


Figure 8: Derived CTMC of LossyFIFO1

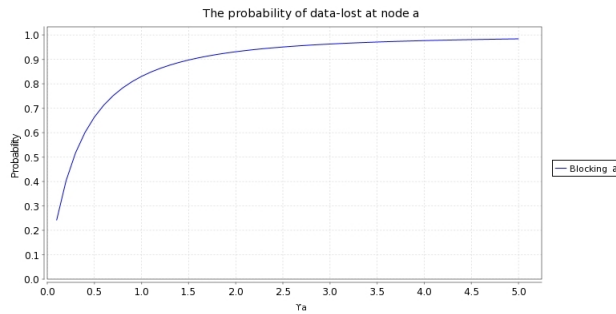


Figure 9: Probability of data lost at node a

5 Related work

The research in formal specification of systems with quantitative aspects encompasses a variety of developments such as Stochastic Process Algebras (SPAs) [6], Stochastic Automata Networks (SANs) [7, 8], and Stochastic Petri nets (SPNs) [9, 10]. SPA is a model for both qualitative and quantitative specification and analysis with a compositional and hierarchical framework. It has algebraic laws (the so-called static laws) and expansion laws which express parallel compositions in terms of SPA operators. In SPA the interpretation of the parallel composition is a vexed one because it allows various interpretations such as Performance Evaluation Process Algebra (PEPA) [11], and Extended Markovian Process Algebra (EMPA) [12, 13]. SPA describes ‘*how*’ each process behaves, while (Stochastic) Reo directly describes ‘*what*’ communication protocols connect and coordinate the processes in a system, in terms of primitive channels and their composition. Therefore, (Stochastic) Reo explicitly models the pure coordination and communication protocols including the impact of real communication networks on software systems and their interactions. Compared to SPA, our approach more naturally leads to a formulation using queueing models.

SPN is widely used for modelling concurrency, synchronization, and precedence, and is conducive to both top-down and bottom-up modelling. Stochastic Reo shares the same properties with SPN and

natively supports composition of synchrony and exclusion together with asynchrony. The topology of connectors in (Stochastic) Reo is inherently dynamic, and it accommodates mobility [22]. Moreover, (Stochastic) Reo supports a liberal notion of channels and is more general than data-flow models and Petri nets, which can be viewed as specialized channel-based models that incorporate certain specific primitive coordination constructs.

SAN consists of a couple of stochastic automata that act independently. Thus, the state of SAN at time t is expressed by the states of each automaton at time t . The concept of a collection of individual automata helps modeling distributed and parallel systems more easily. The interactions in SAN are rather limited to patterns like synchronizing events or operating at different rates. Compared with the SAN approach, the expressiveness of (Stochastic) Reo makes it possible to model different interaction patterns involving both asynchronous and synchronous communications.

Continuous-Time Constraint Automata (CCA) [20] are another stochastic extension of CA that support reasoning about QoS aspects such as expected response times. CCA are close to Interactive Markov Chains (IMCs) [19], that is, they have two types of transitions called *interactive transitions* and *Markovian transitions* for, respectively, the immediate interaction with the environment and internal stochastic behaviors. The stochastic extension in CCA focuses on internal behavior of a connector, but does not take into account the arrivals of I/O requests at the ends of a connector as a stochastic process, which is required for reasoning about the end-to-end QoS of a system.

6 Conclusions and Future work

We introduced Stochastic Reo automata by extending Reo automata with functions that assign stochastic values of arrival rates and processing delay rates to boundary nodes and channels in Stochastic Reo. This model is very compact compared to the existing models, e.g., in [4]. Various formal properties of our model are obtained, reusing the formal justifications of the various properties of Reo automata [3], such as compositionality.

The technical core in this paper shows the complexity of the original problem whence it stems from: derivation of stochastic models for formal analysis of end-to-end QoS properties of systems composed of services/components supplied by disparate providers, in their user environments. This complexity highlights the gross inadequacy of informal, or one-off techniques and emphasizes the importance of formal approaches and sound models that can serve as the basis for automated tools.

Stochastic Reo does not impose any restriction on the distribution of its annotated rates such as the rates for data-arrivals at channel ends or data-flows through channels. However, for translation of stochastic Reo to a homogeneous CTMC model, we considered only the exponential distributions for the rates. As future work, we also want to consider non-exponential distributions by considering phase-type distributions [14] or using Semi-Markov Processes [15] as target models of our translation. A simulation engine [21], already integrated into our toolset, Eclipse Coordination Tools (ECT) [18] environment, supports a wide variety of more general distributions for Stochastic Reo. In our future work, we will consider rewards of a system along with its stochastic behavior as well. Our translation result will thus become a CTMC model with reward information on its transitions and states, and can be fed into a stochastic analysis tool. We will also implement a tool for our translation approach via Stochastic Reo automata. It will be an extension of the existing tools in ECT, for instance, by implementing the synchronization in Stochastic Reo automata. Furthermore, as a case study, we are currently applying our method to an industrial application, the ASK system [16], by modeling the system with Stochastic Reo and analyzing the CTMC derived from its resulting Stochastic Reo automaton model [23].

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