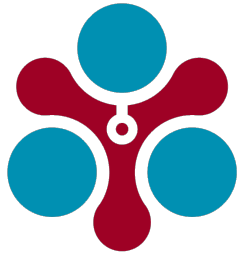
A Personal View of Average-Case Complexity

Russel Impagliazzo
Presented by Bernardo Portela

InfoBlender – 9th of March 2016



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Overview

Motivation

Some context

Five worlds

Computational complexity

"There is a large gap between a problem not being easy and the same problem being difficult"

- Russel Impagliazzo, 1995

Preliminaries: P and NP

P: Polynomial time

The solution can be efficiently computed.

- Linear programming, ordering, ...

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P: Polynomial time

The solution can be efficiently computed.
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NP: Non-deterministic polynomial time

The solution cannot be efficiently computed, but can be efficiently verified.
- Hamiltonian path is NP-complete

$$\mathsf{P} \subseteq \mathsf{NP}$$

Preliminaries: One-wayness and PKC

One-way function

A function that is easy to compute for every input, but hard to invert given the image of a random output.

- E.g. Cryptographic hash functions

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Public-key cryptography

Simplified as "the task of agreeing on a secret with a stranger over an untrusted channel".

Initial Considerations

What would be the implications of solving the problems of computational complexity?

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- Large scale.
- Gauss versus Grouse.

Some assumptions

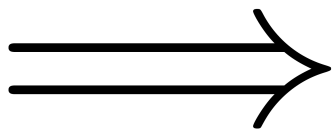
- Undefined computational model.
- Quantitative details ignored.
- Exists implies that everyone knows it.
- If one can run it, everyone can.

Algorithmica

$$P = NP$$

- Grouse fails miserably.
- Computational power allows for extraordinary optimizations.
- Impossible security?

Algorithmica

- Efficient algorithm for an NP-complete problem.  Algorithmica

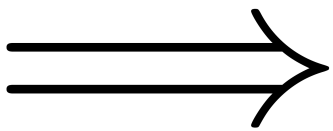
Heuristica

NP hard on worst-case, but feasible on the average-case.

- Grouse fails (less miserably).
- Computational power covers a reduced amount of solutions.
- Impossible security?

Heuristica

- Algorithm for solving NP on average case.
- Lower bound for NP on worst case.



Heuristica

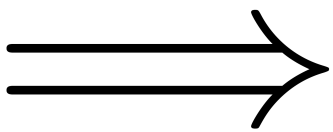
Pessiland

NP hard on average-case. There are no one-way functions.

- "I don't know sir, what's the answer?"
- Optimization via heuristics.
- Problems are useless for cryptography.

Pessiland

- Algorithm for inverting one-way functions.
- Lower bound for NP on average case.



Pessiland

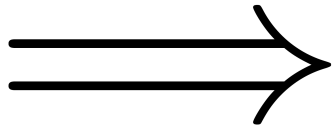
Minicrypt

One-way functions exist. Public-key cryptography impossible.

- The bad guy wins.
- Computation is similar to our world.
- No key agreement, no multiparty computation.

Minicrypt

- No efficient algorithm for inverting one-way functions exists.
- Algorithm for breaking key agreement.



Minicrypt

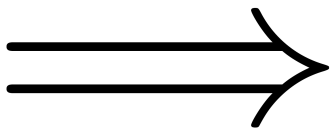
Cryptomania

Public-key cryptography is secure.

- Gauss is utterly defeated.
- Computation is similar to our world.
- Security is similar to our world?

Cryptomania

- Prove a key-agreement as secure.



Cryptomania

Discussion

- The most relevant problems identified in 1995 are still of great relevance in 2016.
- Cryptography and security are always pursuing the middle-ground.
- The worst possibility might not be that bad.

Wrap-up

1. Algorithmica : P = NP
2. Heuristica: NP hard on worst-case, feasible on the average
3. Pessiland: NP hard on average, no one-way functions
4. Minicrypt: One-way functions, no public-key crypto
5. Cryptomania: Public-key crypto

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