# Pure Operation-Based Replicated Data Types*

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October 12, 2017

**Abstract**

Distributed systems designed to serve clients across the world often make use of geo-replication to attain low latency and high availability. Conflict-free Replicated Data Types (CRDTs) allow the design of predictable multi-master replication and support eventual consistency of replicas that are allowed to transiently diverge. CRDTs come in two flavors: state-based, where a state is changed locally and shipped and merged into other replicas; operation-based, where operations are issued locally and reliably causal broadcast to all other replicas. However, the standard definition of op-based CRDTs is very encompassing, allowing even sending the full-state, and thus imposing storage and dissemination overheads as well as blurring the distinction from state-based CRDTs. We introduce pure op-based CRDTs, that can only send operations to other replicas, drawing a clear distinction from state-based ones. Data types with commutative operations can be trivially implemented as pure op-based CRDTs using standard reliable causal delivery; whereas data types having non-commutative operations are implemented using a *PO-Log*, a partially ordered log of operations, and making use of an extended API, i.e., a *Tagged Causal Stable Broadcast (TCSB)*, that provides extra causality information upon delivery and later informs when delivered messages become causally stable, allowing further *PO-Log* compaction. The framework is illustrated by a catalog of pure op-based specifications for classic CRDTs, including counters, multi-value registers, add-wins and remove-wins sets.

## 1 Introduction

Eventual consistency [36] is a relaxed consistency model that is often adopted by large-scale distributed systems [11, 3, 34, 13] where losing availability is normally not an option, whereas delayed consistency is acceptable. In eventually consistent systems, data replicas are allowed to temporarily diverge, provided that they can eventually be reconciled into a common consistent state. Reconciliation (or merging) used to be error-prone, being application-dependent, until new data-type-dependent models like the Conflict-free Replicated Data Types (CRDTs) [31, 33] were introduced. CRDTs allow both researchers and practitioners to design correct replicated data types that are always available, and are guaranteed to eventually converge once all operations are known at replicas. The concept of CRDT has

*The work presented was partially supported by EU FP7 SyncFree project (609551), EU H2020 LightKone project (732505), and SMILES line in project TEC4Growth (NORTE-01-0145-FEDER-000020.

been deployed in practice by industry, initially in Key-Value Stores [11], and have since been ported to multiple languages and production platforms.

CRDTs support two complementary designs: operation-based (or simply, op-based) and state-based. As the name suggests, the former are based on dissemination of operations and the later on shipping state that results from locally applied operations. In op-based designs [21, 33], the execution of an operation is done in two phases: *prepare* and *effect*. The former is performed only on the local replica and looks at the operation and current state to produce a message that aims to represent the operation, which is then shipped to all replicas. Once received, the representation of the operation is applied remotely using *effect*. Different replicas are guaranteed to converge as long as messages are disseminated through a reliable causal broadcast messaging middleware [8], and *effect* is designed to be commutative for concurrent operations. On the other hand, in a state-based design [4, 33], an operation is only executed on the local replica state. A replica propagates its local changes to other replicas through shipping its entire state. A received state is incorporated with the local state via a *merge* function that, deterministically, reconciles the *merged* states. To maintain convergence, *merge* is defined as a join: a least upper bound over a join-semi-lattice [4, 33]. Typically, state-based CRDTs support ad hoc dissemination of states and can handle duplicate and out-of-order delivery of messages without breaking causal consistency; however, they impose complex state designs and store extra meta-data. On the other hand, in the systems where the message dissemination layer guarantees reliable causal broadcast, operation-based CRDTs have more advantages as they can allow for simpler implementations, concise replica state, and smaller messages.

In standard op-based CRDTs the designer is given much freedom in defining *prepare*, namely using the state in an arbitrary way. This is needed to have the *effect*s of concurrently invoked data-type operations commute, and thus provide replica convergence despite the absence of causality information in current causal delivery APIs. This forces current op-based designs to include causality information in the state to be used in *prepare*, sent in messages, and subsequently used in *effect*. The designer ends up intervening in many components (the state, *prepare*, *effect*, and *query* functions) in an ad hoc way. This can result in large complex state structures and also large messages.

Currently, a *prepare* not only builds messages that duplicate the information already present in the middleware (even if it is not currently made available), but causality meta-data is often incorporated in the object state, hence, reusing design choices similar to those used in state-based approaches. Such designs impose larger state size and do not fully exploit causal delivery information. This freedom in current op-based designs is against the spirit of 'sending operations', and leads to confusion with the state-based approach. Indeed, in the current op-based framework, a *prepare* can return the full state, and an *effect* can do a full state-merge (which mimics a state-based CRDT) [4, 33].

We believe that the above weaknesses can be avoided if the causality meta-data can be provided by the messaging middleware. Causal broadcast implementations already possess that information internally, but it is not exposed to clients. In this paper we propose and exploit such an extended API to achieve both simplicity and efficiency in defining op-based CRDTs.

We introduce a *Pure* Op-Based CRDT framework, in which *prepare* cannot inspect the state, being limited to returning the operation (including potential parameters). The entire logic of executing the operation in each replica is delegated to *effect*, which is also made generic (i.e., not data type dependent). For pure op-based CRDTs, we propose that the object state is a *partially ordered log of operations – a PO-Log*. Causality information is provided by an extended messaging API: *Tagged Causal Stable Broadcast* (TCSB). We use this information to preserve convergence and also design compact and efficient CRDTs through a *semantically based PO-Log compaction* framework, which makes use of a data type-specific *obsolescence* relation, defined over timestamp-operation pairs.

Furthermore, we propose an extension that improves the design and implementation of op-based CRDTs through decomposing the state into two components: a PO-Log (as before), and a causality-stripped-component which, in many cases, will be simply a standard sequential data type. For this, we introduce the notion of *causal stability*, to be provided by the TCSB middleware. The idea is that operations are kept only transiently in the PO-Log, but once they become causally stable, causality meta-data is stripped, and the operations are stored in the sequential data type. This reduces the storage overhead to a level that was never achieved before in CRDTs, neither state-based nor op-based.

# 2 System Model and Notations

The system is composed of a fixed set of nodes, each is associated with a globally unique identifier in a set $I$. Nodes execute operations at different speeds and communicate using asynchronous message passing. Messages can be lost, reordered, or duplicated, and the system can experience arbitrary, but transient, network partitions. To ensure exactly-once delivery, message sending is abstracted by a reliable causal broadcast [9, 17]. A node can fail by crashing and can recover later on. Upon recovery, the last durable state of a node is assumed to be intact (not destroyed). We do not consider Byzantine or malicious faults.

For presentation purposes, and without loss of generality, we consider a single data type instance that is fully replicated at each node. All replicas (of a data type) are initially equivalent. Once a data type operation is locally applied on a replica, the latter can diverge from other replicas, but replicas may eventually convergence as operations arrive everywhere. The local application of an operation is atomic on each replica (i.e., upon a crash, no state 'in the middle of an operation' can be seen in the durable state).

## 2.1 Definitions and Notations

$\Sigma$ denotes the type of the state. $\mathcal{P}(V)$ denotes a *power set* (the set of all subsets of $V$), where $V$ is a set of any type. The initial state of a replica $i$ is denoted by $\sigma_i^0 \in \Sigma$. Operations belong to a set $O$ and can include arguments, in which case they are surrounded by square brackets, e.g., $\mathsf{inc}$ and $[\mathsf{add}, v]$. On the other hand, the notation $o[j]$ refers to the $j^{th}$ element in a list that comprises the operation name and subsequent arguments (analogous to the $argv[]$ used in several programming languages); in particular, $o[0]$ refers to the operation name. We use total functions $K \rightarrow V$ and maps (partial functions) $K \hookrightarrow V$ from keys to values, both represented as sets of pairs $(k, v)$. Given a function $m$, the notation $m\{k \mapsto v\}$ maps a specific key $k$ to $v$, and behaves like $m$ on other keys.

# 3 Background

By tracking divergence, conflict-free replicated data-types allow local operation even when no communication is possible. We have seen that CRDTs can be approached by state-based[1]

[1]For more information about state-based CRDTs and the optimized variant Delta-State CRDTs, the reader may refer to [31, 33] and [1, 24].

or op-based frameworks, and both approaches share the same high-availability properties and resiliency to partitions. Onwards, this paper will focus on advancing the definition and possibilities of the operation-based approach.

### 3.1 Operation-Based CRDTs

In op-based CRDTs [31, 33], each replica maintains a local state, and is subject to clients *query* and *update* operations that are executed locally as soon as they arrive. In particular, update operations that are received through clients are disseminated to other replicas, via a reliable causal broadcast middleware, in a form of operation name, arguments (if exist), and possibly some meta-data [31, 33]. Once all replicas receive and execute all update operations, they eventually converge to a single state as long as concurrent update operations commute, i.e. produce the same effect if delivered in different orders. Operation dissemination must be reliable, and most data-types require broadcast delivery to respect causal order. This also ensures that resulting states are causally consistent.

**Algorithm 1** Distributed algorithm at node $i$ showing the interplay between a standard reliable causal broadcast API ($\mathsf{cbcast}$ and $\mathsf{cdeliver}$) and op-based CRDTs ($\mathsf{prepare}$ and $\mathsf{effect}$).

**state:**
  $\sigma_i \in \Sigma$
**on** $\mathsf{operation}_i(o)$:
  $m := \mathsf{prepare}_i(o, \sigma_i)$
  $\mathsf{cbcast}_i(m)$
**on** $\mathsf{cdeliver}_i(m)$:
  $\sigma_i := \mathsf{effect}_i(m, \sigma_i)$

| | |
|---|---|
| $\Sigma$ | : State type, $\sigma_i$ is an instance |
| $\mathsf{prepare}_i(o, \sigma_i)$ | : Prepares a message $m$ given an operation $o$ |
| $\mathsf{effect}_i(m, \sigma_i)$ | : Applies a *prepared* message $m$ on a state |
| $\mathsf{eval}_i(q, \sigma_i)$ | : Read-only evaluation of query $q$ on a state |

Figure 1: The general scheme of an op-based CRDT.

More specifically, the typical scheme of op-based CRDT is depicted in Figure 1, while Algorithm 1 depicts the interplay between the reliable causal broadcast middleware and the CRDT. When an update operation $o$ is issued at some node $i$ having state $\sigma_i$, the function $\mathsf{prepare}_i(o, \sigma_i)$ is called returning a message $m$. This message $m$ is then broadcast by calling $\mathsf{cbcast}_i(m)$ that is provided by the reliable causal broadcast API. Once $m$ is delivered via $\mathsf{cdeliver}_j(m)$ at each destination node $j$ (including $i$ itself), $\mathsf{effect}_j(m, \sigma_j)$ is called, returning a new replica state $\sigma'_j$. For each node that broadcasts a given operation, the broadcast event, the corresponding local delivery, and the $\mathsf{effect}$ on the local state are executed atomically. When a query operation $q$ is issued, $\mathsf{eval}_i(q, \sigma_i)$ is invoked, and no corresponding broadcast occurs. $\mathsf{eval}$ takes the query and the state as input and returns a result, leaving the state unchanged. We further explain this through the following simple example.

*Example on Op-Based Counters:* Figure 2a represents an op-based increment-only counter. The state represents an integer that is initialized to 0. The $\mathsf{prepare}$ function only returns

$m = \mathsf{inc}$ for an invoked inc operation by a client. The middleware disseminates $m$ to all replicas, whereas, effect increments the counter state at each replica. The query operation value simply returns the value of the counter.

### 3.2 The case for a new op-based CRDT model

The above example on counters is useful to explain the op-based CRDT model, but it falsely gives the impression that the CRDT design is generally straightforward and lightweight. Unfortunately, this is not the case for more complex data types where operations are not commutative, e.g., in sets, maps, etc. For instance, consider the *Add-Wins Set* design shown in Figure 2b and having the following semantics: an add operation prevails if it is concurrent with a remove (an alternative remove-wins design is shown later). Since add and remove are not commutative operations, the causal order of their execution must be ensured to guarantee that different replicas converge. Capturing this information leads to a more complex design through adding extra meta-data to the data type state (see $\Sigma$) as well as to the exchanged messages (see the output of prepare).

Indeed, after analyzing several op-based data types introduced in [31, 33], we observed three general trends (we discuss these in details in Section 9): first, the entire design is data-type-dependent since all the components (i.e., $\Sigma, \mathsf{prepare}, \mathsf{effect}$, and eval) can change per data type; second, designing (or understanding) an efficient data type becomes cumbersome; and third, this design induces a substantial communication overhead since prepare returns additional meta-data to be sent (along with the operation name and parameters). The pure op-based CRDT model that we introduce in this paper tries to address these challenges.

## 4 Pure Op-based CRDTs

In this section, we introduce *pure op-based CRDTs*. We show that data types whose operations are commutative are natively "pure"; whereas for other data types to be pure, they need to leverage some information channeled through the dissemination middleware.

**Definition 4.1 (Pure op-based CRDT)** *An op-based CRDT is* pure *if disseminated messages contain only the operation and its potential arguments.*

Therefore, considering the general op-based CRDT scheme introduced in Figure 1, and given an operation $o$ and a state $\sigma$, prepare is always defined as:

$$\mathsf{prepare}(o, \sigma) = o$$

In addition to the generic nature of this property across different data types, at least two other benefits can be identified. First, prepare cannot build an arbitrary message depending on the current state which avoids introducing design or implementation bugs. Second, the operation can immediately be broadcast without even reading the replica state $\sigma$. (Indeed, the parameter $\sigma$ of $\mathsf{prepare}(o, \sigma)$ can be omitted in pure op-based CRDTs, but we keep it for consistency and clarity.). This has good performance implications on the sender side as the access to persistent storage and serialization/deserialization are avoided.

Back to the data type examples described in Figure 2, we notice that the counter data type is pure op-based since $\mathsf{prepare} = \mathsf{inc}$. On the contrary, the Add-Wins Set in Figure 2b is not pure since a prepare builds a set of triples present in the current state, to be used by other

$$
\begin{array}{rcl}
\Sigma = \mathbb{N} & & \sigma_i^0 = 0 \\
\mathsf{prepare}_i(\mathsf{inc}, n) & = & \mathsf{inc} \\
\mathsf{effect}_i(\mathsf{inc}, n) & = & n + 1 \\
\mathsf{eval}_i(\mathsf{value}, n) & = & n
\end{array}
$$

(a) Op-based increment-only counter.

$$
\begin{array}{rcl}
\Sigma = \mathbb{N} \times \mathcal{P}(I \times \mathbb{N} \times V) & & \sigma_i^0 = (0, \{\}) \\
\mathsf{prepare}_i([\mathsf{add}, v], (n, s)) & = & [\mathsf{add}, v, i, n + 1] \\
\mathsf{effect}_i([\mathsf{add}, v, i', n'], (n, s)) & = & (n' \textbf{ if } i = i' \textbf{ otherwise } n \\
& & , s \cup \{(v, i', n')\}) \\
\mathsf{prepare}_i[\mathsf{remove}, v], (n, s)) & = & [\mathsf{remove}, \{(v', i', n') \in s \mid v' = v\}] \\
\mathsf{effect}_i([\mathsf{remove}, r], (n, s)) & = & (n, s \setminus r) \\
\mathsf{eval}_i(\mathsf{elems}, (n, s)) & = & \{v \mid (v, i', n') \in s\}
\end{array}
$$

(b) Op-based Add-Wins Set (embeding causality)

Figure 2: An example of two standard op-based CRDTs showing their design complexity (when causality is needed) and their data-type-dependent nature.

replicas when performing effect (either for add or for rmv). To understand this difference, we make a distinction between two categories: commutative and non-commutative data types.

**Definition 4.2 (Commutative data type)** *A concurrent data type is commutative iff:*

1. *for any operations $f$ and $g$, their sequential invocation commutes: $f(g(\sigma)) = g(f(\sigma))$, and*
2. *all concurrent invocations are defined as equivalent to some linearization.*

While the first property removes the ordering overhead of operations (as no ordering assumptions are required), the second property makes the pure op-based design of a commutative data type straightforward as we explain next.

## 4.1 Pure designs for commutative data types

Commutative data types can natively be designed as pure op-based CRDTs on top of a standard reliable causal broadcast [6]. The reason is that commutative data types reflect a *principle of permutation equivalence* [5]: if all sequential permutations of updates lead to equivalent states, then it should also hold that concurrent executions of the updates lead to equivalent states. As the extension to concurrent scenarios follows directly from their sequential definition (with no room for design choices), commutative data types can have a standard sequential specification and design. Consequently, a pure op-based CRDT

$$
\begin{array}{rcl}
\Sigma = \mathbb{N} & & \sigma_i^0 = 0 \\
\mathsf{prepare}_i(o, \sigma_i) & = & o \quad (\mathsf{inc} \text{ or } \mathsf{dec}) \\
\mathsf{effect}_i(\mathsf{inc}, n) & = & n + 1 \\
\mathsf{effect}_i(\mathsf{dec}, n) & = & n - 1 \\
\mathsf{eval}_i(\mathsf{value}, n) & = & n
\end{array}
$$

(a) Pure PN-Counter

$$
\begin{array}{rcl}
\Sigma = \mathcal{P}(V) & & \sigma_i^0 = \{\} \\
\mathsf{prepare}_i(o, \sigma_i) & = & [\mathsf{add}, v] \\
\mathsf{effect}_i([\mathsf{add}, v], s) & = & s \cup \{v\} \\
\mathsf{eval}_i(\mathsf{elems}, s) & = & s
\end{array}
$$

(b) Pure Grow-Only Set

Figure 3: Pure op-based CRDTs for commutative data types.

design becomes trivial: since a message returned from prepare, and containing the operation name and arguments, will arrive exactly once at each replica (through the reliable causal broadcast), it is enough to have effect only apply the received operation to the standard sequential data type state, i.e., defining for any operation $o$:

$$\mathsf{effect}(o, \sigma) = o(\sigma)$$

*Examples on commutative data types:* Two examples are presented in Figure 3: a PN-Counter with inc and dec operations, and a Grow-only Set (G-Set) with add operation. Both cases use a standard sequential data type for the replica state $\Sigma$, and applying effect is just invoking the corresponding standard operation ($+, -, \cup$, etc.) to the sequential data type. Both examples exploit commutativity and rely on the exactly-once delivery, leading to a trivial pure design.

## 4.2 The challenge of non-commutative data types

In the case where the operations of a data type are not commutative, e.g., add and rmv in a set ($\mathsf{add}(v, \mathsf{rmv}(v, s)) \neq \mathsf{rmv}(v, \mathsf{add}(v, s))$), an effect cannot simply apply the operations over a sequential data type as done in the previous section: $o(\sigma)$. This is due to two reasons, that we explain below, on which we base our extended model.

The first reason is that the messages corresponding to concurrent operations can be delivered in different orders on different replicas. Since the operations do not commute (even if the semantics of concurrent invocations can be defined as equivalent to some linearization of those operations), simply applying them in different orders on different replicas makes these replicas diverge. Therefore, convergence is only guaranteed if effect is commutative, and therefore, cannot be defined directly as operations that are not commutative themselves (e.g., simple add and remove operations). To address this point, effect must have partial-ordering knowledge about operations and a corresponding logic in execution. While the latter suggests changing the design of effect, the former may require exchanging some ordering meta-data. However, since such information about the partial order is already present in the meta-data of the causal delivery middleware, we propose to extend the API of the middleware (presented in Section 5), which we call *Tagged Causal Stable Broadcast* (TCSB), to leverage this meta-data that can then be used in designing general non-commutative pure op-based CRDTs.

The other reason is that, indeed, there are useful concurrent data types in which the outcomes of concurrent executions are not equivalent (on purpose) to some linearization. The best example is the Multi-Value Register (MVRegister) that is used in Amazon's Dynamo system to model the shopping cart [13]. In MVRegister, invoking a read operation following two concurrent writes in its causal past would return a set with *both* values written. This behavior does not arise under a sequential specification of a register, and the state must retain concurrent operations. We address this challenge by proposing (in Section 6) the type of a CRDT (i.e., $\Sigma$) to be a partially-ordered log of operations provided that $\mathsf{effect}$ understands how to execute these operations, thanks to the TCSB meta-data.

# 5 Tagged Causal Stable Broadcast (TCSB)

Reliable Causal Broadcast (RCB) is a prominent dissemination abstraction that is often used in AP systems (in the CAP [15] context) to ensure causal delivery of messages, being the strongest consistency model in always-available systems that eventually converges [22, 2]. A common implementation strategy for a reliable causal broadcast service [30] is to assign a vector clock to each message broadcast, and use the causality information in the vector clock to decide at each destination when a message can be delivered. If a message arrives at a given destination before causally preceding messages have been delivered, the service delays delivery of that message until those messages arrive and are delivered.

When messages are delivered to the application layer, in a sequence that is consistent with causality, there is some loss of information when using standard RCB middleware. Messages that the middleware knows be concurrent, are delivered in order and the application is unaware of that concurrency. Also the middleware can often determine that, for a given delivered message, that no more messages concurrent to it are to be delivered. This information, that can be used to optimize message processing by the application, is also not conveyed. Next we show how to expose this ordering information and how to provide a causal stability oracle that informs on stable messages, those with no further concurrent deliveries.

## 5.1 Exposing Ordering Information

Reliable Causal Broadcast (RCB) uses internally a logical timestamp $t$ (e.g., a vector clock) to order messages. We propose to simply expose this information to the upper layers. Technically, the $\mathsf{cdeliver}(m)$ middleware API (used in Algo. 1) will now look like this: $\mathsf{tcdeliver}(t, m)$; meaning that, the calling process will receive the message $m$ as well as its associated timestamp $t$. Having a timestamp provided by the middleware allows the application logic to access causality information and avoids duplicating it in the datatype logic, avoiding the complexity in Figure 2b and simplifying the message payload.

## 5.2 Causal Stability

In concurrent data types, the order information provided by timestamps may only be needed for an operation as long as its concurrent operations are being delivered or expected. However, in our experience, this information is useless once no concurrent operations are expected for a given operation, which we then call a "causally stable" operation and thus, it makes sense to get rid of this extra meta-data. Consequently, we propose another extension to the RCB middleware in order to provide *causal stability* information to the upper layers. We first define causal stability.

**Definition 5.1 (Causal Stability)** *A timestamp $\tau$, and a corresponding message, is* causally stable *at node $i$ when all messages subsequently delivered at $i$ will have timestamp $t > \tau$.*

This implies that no message with a timestamp $t$ concurrent with $\tau$ can be delivered at $i$ when $\tau$ is causally stable at $i$. This notion differs from the classical message stability, in [7], in which a message is considered stable if it has been received by all nodes. Our definition is stronger, as we also require that no further concurrent messages may be delivered. Our definition is about delivered (as opposed to received) messages, and is per-node (some $m$ may be causally stable at some node $i$ but not at some node $j$), not global. It is similar to the definition in [31], but we consider important to use a different name, to avoid confusion with classic message stability.

The TCSB middleware can offer this *causal stability* information through extending its API with $\mathsf{tcstable}_i(\tau)$ which informs the upper layers that message with timestamp $\tau$ is now known to be causally stable. This can be done by each node having a causal stability oracle which conservatively detects causal stability according to local knowledge. Each node $i$ can check if timestamp $\tau$ is causally stable at $i$ by verifying if a message with timestamp $t > \tau$ has already been delivered at $i$ from every other node $j$ in the set of nodes $I$. More formally, if $\mathsf{deliv}_i()$ returns the set of messages that have been delivered at node $i$ and $\mathsf{src}(t)$ denotes the node from where the message corresponding to $t$ has been issued:

$$\mathsf{tcstable}_i(\tau) \textbf{ if } \forall j \in I \setminus \{i\} \cdot \exists t \in \mathsf{deliv}_i() \cdot \mathsf{src}(t) = j \wedge \tau < t.$$

One possible implementation of a TCSB causal stability oracle uses a strategy similar to Roh's RGA tombstone deletion algorithm [27]. Each node $i$ keeps a local map $L_i$ (from $I$ to $T$) with the last vector clock timestamp, delivered locally, from each other node. We define an auxiliary function $\mathsf{low}$ that gives the greatest lower bound on messages issued at $j$ delivered at each other node:

$$\mathsf{low}(L, j) \doteq \min(\{L(k)(j) \mid k \in \mathsf{dom}(m)\}).$$

For instance, if $\mathsf{low}(L_i, j) = 4$, we know at node $i$ that all other nodes have delivered at least message number 4 from node $j$. Using this function we can now define a sufficient condition for the causal stability oracle. A timestamp $\tau$ is causally stable at node $i$ if

$$\tau(\mathsf{src}(\tau)) \leq \mathsf{low}(L_i, \mathsf{src}(\tau)).$$

This ensures at $i$ that message $\tau$ was already delivered at all nodes and that each node issued a message after delivering $\tau$, that is already delivered at $i$. Since any messages concurrent to $\tau$ from any node $k$ must have been issued prior to $\tau$ being delivery at $k$ then, due to causal delivery, they must have also been delivered at node $i$.

# 6 Pure CRDTs Based on Partially Ordered Logs (PO-Log)

We introduce a new framework for designing pure op-based CRDTs of non-commutative data types. Driven by the challenges discussed in Section 4.2, the framework supports concurrent data types through the use of a Partial Ordered Log (PO-Log for short) which retains all invoked operations together with their timestamps. Being "pure", i.e., the $\mathsf{prepare}$ in framework shall only have the name of the operation and potential arguments, the timestamp

$$
\begin{array}{rcl}
\Sigma = T \hookrightarrow O & & \sigma_i^0 = \{\} \\
\mathsf{prepare}_i(o, s) & = & o \\
\mathsf{effect}_i(o, t, s) & = & s \cup \{(t, o)\} \\
\mathsf{eval}_i(q, s) & = & \text{[datatype-specific query function]}
\end{array}
$$

Figure 4: PO-Log based reference implementation for pure op-based CRDTs

that is stored in the PO-Log is directly fed by the underlying TCSB middleware introduced above. Figure 4 depicts a reference design of a pure CRDT, whereas Algorithm 2 describes its interaction with the TCSB middleware. We describes the framework in this section showing that it is easy to understand and generic. The framework is however not practical without the optimizations we introduce in the subsequent sections — due to the ever-growing PO-Log.

The challenges leveraged in Section 4.2 can be summarized by two points. The first is that there is a real need to support data types that are concurrent in practice. Consequently, we choose to retain all operations in a partially ordered log (PO-Log) that can be implemented as a map from timestamps to operations: $\Sigma := T \hookrightarrow O$ as shown in Figure 4. This brings two main benefits: (1) a PO-Log can retain concurrent operations, and (2) storing operations as black-boxes makes the state type generic across all data types. The timestamps offered by the TCSB middleware through $\mathsf{tcdeliver}$ (as in Algorithm 2) are crucial to keep information about concurrent operations, not trying to impose a local total-order over them, contrary to a classic sequential log.

The second challenge, i.e., how to design the $\mathsf{effect}$, is interestingly made easy using the PO-Log. In fact, we can now obtain a universal data-type-independent definition of $\mathsf{effect}$:

$$
\mathsf{effect}(o, t, s) = s \cup \{(t, o)\},
$$

which associates a timestamp to the corresponding operation and joins it to the PO-Log via the set union $\cup$, which is commutative, therefore making the $\mathsf{effect}$ commutative as needed.

In this novel framework, it is easy to notice that all the components, but one, of the data type design are the same regardless of the data type. This significantly reduces the burden of implementation on developers as well as the chances of introducing new bugs. Only the query functions, defined in $\mathsf{eval}$, will need to be data-type-specific according to the desired semantics; however, their definition over the PO-Log will typically be a direct transposition of their specification, which is fairly easy to define as we show next in the examples.

### 6.1 Example CRDTs

We exemplify the use and simplicity of the above framework through two examples: an Add-Wins Set (AWSet) and a Multi-Value Register (MVRegister). In a nutshell, the semantics of AWSet give priority for $\mathsf{add}$ operation over a $\mathsf{remove}$ if they are concurrent; whereas, an MVRegister retains all concurrently written values and leaves the decision to the application to choose the desired value. These semantics are explained in details in Section 8.

Figures 5 and 6 show two pure PO-Log designs for AWSet and MVRegister, respectively, using the framework presented in Fig. 4. It is interesting to notice that the pure CRDT

**Algorithm 2** Distributed algorithm for node $i$ showing the interaction between the Tagged Causal Stable Broadcast (TCSB) middleware and a Pure CRDT.

**state:**
  $\sigma_i \in \Sigma$
**on** $\mathsf{operation}_i(o)$:
  $m := \mathsf{prepare}_i(o, \sigma_i)$
  $\mathsf{tcbcast}_i(m)$
**on** $\mathsf{tcdeliver}_i(m, t)$:
  $\sigma_i := \mathsf{effect}_i(m, t, \sigma_i)$
**on** $\mathsf{tcstable}_i(t)$:
  $\sigma_i := \mathsf{stable}_i(t, \sigma_i)$

$$
\begin{array}{rcll}
\Sigma = T \hookrightarrow O & & \sigma_i^0 = \{\} & \\
\mathsf{prepare}_i(o, s) & = & o & (o \text{ is } [\mathsf{add}, v] \text{ or } [\mathsf{rmv}, v]) \\
\mathsf{effect}_i(o, t, s) & = & s \cup \{(t, o)\} & \\
\mathsf{eval}_i(\mathsf{elems}, s) & = & \{v \mid (t, [\mathsf{add}, v]) \in s \wedge \nexists (t', [\mathsf{rmv}, v]) \in s \cdot t < t'\} &
\end{array}
$$

Figure 5: PO-Log based Add-Wins Set (AWSet).

design of both data types is exactly the same considering the state $\Sigma$, prepare, and also effect. The only data-type-specific function is the eval query function which defines the behavior of the data types in both CRDTs. In AWSet (Fig. 5), the values reported to be in the set are those corresponding to the add operations in the PO-Log having no remove in their causal future. Whereas, in MVRegister (Fig. 6), a read reports the set of all distinct concurrently written values, via wr, that have not been overwritten.

Although these designs are not realistic to be used in practice, since the state size in each replica grows linearly with the number of operations, they represent a starting point from which actual efficient designs can be derived by semantic PO-Log compaction, as we show in the rest of the paper. In addition, these designs are theoretically relevant as they provide a clear description of the concurrent semantics of the replicated data type, and therefore can be used as a base-line to compare with other designs. This is made possible through capturing the partial ordered set of all operations delivered to each replica[2].

# 7 Semantic PO-Log Compaction

In this section, we show how PO-Log based CRDTs can be made practical by performing PO-Log compaction, and thus extend the introduced framework. The purpose of compaction is to reduce the space and computation overheads through pruning the PO-Log in a lossless manner. We do this through two main mechanisms: *causal redundancy* which once an operation is delivered, one may prune that very operation or/and an already existing PO-Log operation, whichever is deemed redundant according to the data type semantics; and

[2]A similar approach to express data type semantics is found in [10] when relating to the *visibility* relation.

$$
\begin{array}{rcll}
\Sigma = T \hookrightarrow O & & \sigma_i^0 = \{\} & \\
\mathsf{prepare}_i(o, s) & = & o & (o \text{ is } [\mathsf{wr}, v]) \\
\mathsf{effect}_i(o, t, s) & = & s \cup \{(t, o)\} & \\
\mathsf{eval}_i(\mathsf{rd}, s) & = & \{v \mid (t, [\mathsf{wr}, v]) \in s \wedge \nexists (t', [\mathsf{wr}, v']) \in s \cdot t < t'\} &
\end{array}
$$

Figure 6: PO-Log based Multi-Value register (MVRegister)

$$
\begin{array}{rcl}
\Sigma = T \hookrightarrow O & & \sigma_i^0 = \{\} \\
\mathsf{prepare}_i(o, s) & = & o \\
\mathsf{effect}_i(o, t, s) & = & \{(t, o) \mid (t, o) \not\mathsf{R}\ s\} \cup \{x \in s \mid x \not\mathsf{R}_0\ (t, o) \\
 & & \wedge\ (t, o)\ \mathsf{R}\ s \vee x \not\mathsf{R}_1\ (t, o) \wedge (t, o) \not\mathsf{R}\ s\} \\
\mathsf{R}, \mathsf{R}_0, \mathsf{R}_1 & = & [\text{datatype-specific redundancy relations}] \\
\mathsf{eval}_i(q, s) & = & [\text{datatype-specific query function}]
\end{array}
$$

Figure 7: Generic framework with PO-Log compaction by causal redundancy

*causal stabilization* which once some operation in the PO-Log becomes causally stable, it discards the operation timestamp and possibly removes it or even some other operations.

## 7.1 Causal Redundancy

The first mechanism prunes the PO-Log once an operation is causally delivered in the $\mathsf{effect}$. The aim is to keep the smallest number of PO-Log operations such that all queries return the same result as if the full PO-Log was present. In particular, this method discards operations from the PO-Log if they can be removed without impacting the output of query operations. Since this pruning is based on the semantics of a data type, it is obviously data-type-dependent; however, we opt to call it causal redundancy since pruning is mainly driven by causality.

In Fig. 7, we extend the generic framework which now includes the PO-Log, $\mathsf{prepare}$, and a more sophisticated generic $\mathsf{effect}$. The $\mathsf{effect}$ can now prune the PO-Log through three data-type-specific relations that together define causal redundancy. This has the advantage of keeping the framework generic, with the same $\mathsf{effect}$ for all data types, while delegating data-type-specific behavior to mere relations (and not arbitrary procedures).

The $\mathsf{R}$ relation, in Figure 7, defines whether the delivered operation is itself redundant and does not need to be added itself to the PO-Log. In most cases, as we will see later, this can be decided by looking only at the delivered operation itself, regardless of the current PO-Log and, thus, a unary $\mathsf{R}$ would be enough; however, in other cases, this depends on the PO-Log; therefore, $\mathsf{R}$ is a binary relation between the operation and the PO-Log.

The other two relations, $\mathsf{R}_0$ and $\mathsf{R}_1$, define which operations in the current PO-Log become redundant given the delivery of the new operation. For some data types this decision depends on whether the delivered operation is added to the PO-Log or whether it is itself redundant: we may be able to discard some operation if the new one is kept, but not if it

$$
\begin{array}{rcl}
(t, o)\ \mathsf{R}\ s & \iff & o[0] = \mathsf{clear} \vee \mathsf{rmv} \\
(t', o')\ \mathsf{R\_}\ (t, o) & \iff & t' < t \wedge (o[0] = \mathsf{clear} \vee o[1] = o'[1]) \\
\mathsf{eval}_i(\mathsf{elems}, s) & = & \{v \mid (\_, [\mathsf{add}, v]) \in s\}
\end{array}
$$

Figure 8: Add-Wins set with PO-Log compaction. $o[0]$ denotes the operation name and $o[1]$ denotes its first argument.

is itself discarded. Having these two relations makes the framework expressive enough for such data-types (which are fortunately not numerous): $\mathsf{R}_0$ is used when the new arrival is discarded being redundant, and $\mathsf{R}_1$ if it is added to the PO-Log. For most data types, these relations will be equal, i.e., $\mathsf{R}_0$=$\mathsf{R}_1$, and we define a single $\mathsf{R\_}$ relation to denote both $\mathsf{R}_0$ and $\mathsf{R}_1$. As we will see, the PO-Log compaction resulting from these relations will provide invariants over the PO-Log which allow simplified query functions that give the same result over the compact PO-Log as the original query functions over the full PO-Log.

Having explained the roles of $\mathsf{R}$, $\mathsf{R}_0$, and $\mathsf{R}_1$, the behavior of effect in Figure 7 becomes as follows: a newly delivered operation $(t, o)$ is added to the PO-Log if it is not redundant by the PO-Log operations, as the first clause says: $\{(t, o) \mid (t, o)\ \not\mathsf{R}\ s\}$. The remaining clause says that an existing operation $x$ in the PO-Log is removed if it is made redundant by $(t, o)$. Since, the behavior is sometimes different depending on whether $(t, o)$ has been discarded or not, the two clauses, separated by $\vee$, distinguishes these two cases.

### 7.1.1 Example CRDTs

We apply this PO-Log compaction to the previous examples presented in Figures 5 and 6, extended with a clear operation that removes all values; the corresponding compact designs are presented in Figures 8 and 9, respectively. In the latter two, we only mention the data-type-specific functions since $\Sigma$, prepare, and effect are generic. In these examples, the left side of the relations $\mathsf{R}$ and $\mathsf{R\_}$ are made redundant by the right side. On the other hand, the notation $o[i]$ refers to the $i^{th}$ element in a list that comprises the operation name and subsequent arguments (analogous to the $argv[]$ used in several programming languages); in particular, $o[0]$ refers to the operation name.

In the AWSet design in Figure 8, the first clause says that clear and remove operations are always made redundant by (and thus not stored in) the PO-Log. The second clause specifies that an existing operation $o'$ in the PO-Log is redundant (and thus pruned) if the newly arrived operation $o$ is strictly in its causal future, provided that $o$ is either clear or otherwise (e.g., add) tries to modify the same value (i.e., set item) in the set (in which case it is clearly redundant). These relations will actually result in a PO-Log that is deprived of any rmv or clear operations; consequently, the rd operation will simply return all the values in the PO-Log.

The second example is the MVRegister, presented in Figure 9. The first clause specifies that a clear operation is always redundant and must not be added to PO-Log. The second clause says that an existing operation $o'$ is made redundant by $o$ if the latter is strictly in its causal future; otherwise all concurrent wr operations are kept in the PO-Log. The rd operation is thus very simple: it returns all the present values in the PO-Log.

$$
\begin{array}{rcl}
(t, o) \ \mathsf{R} \ s & \iff & o[0] = \mathsf{clear} \\
(t', o') \ \mathsf{R\_} \ (t, o) & \iff & t' < t \\
\mathsf{eval}_i(\mathsf{rd}, s) & = & \{v \mid (\_, [\mathsf{wr}, v]) \in s\}
\end{array}
$$

Figure 9: Multi-Value register with PO-Log compaction

## 7.2 Causal Stabilization

*Causal stabilization* is the second PO-Log compaction mechanism; it exploits *causal stability* information to discard timestamp meta-data for the elements that become *causally stable*, and may allow some elements to be entirely removed from the PO-Log.

### 7.2.1 Discarding stable timestamps

This mechanism discards the causally stable timestamps in the PO-Log. The motivation is that the effect function makes use of the relations $\mathsf{R}$, $\mathsf{R}_0$, and $\mathsf{R}_1$ to compare $(t', o')$ elements in the PO-Log with newly delivered operations, but never compares PO-Log elements among themselves. Our observation is that most of the meta-data in the PO-Log is not needed when incoming operations are no longer concurrent. Considering the definition of causal stability in Section 5: if some pair $(t', o')$ is in the PO-Log, with $t'$ a causally stable timestamp, all future deliveries $(t, o)$ used in effect will be in its causal future, i.e., $t' < t$. This means that what is important is to know if this comparison $t' < t$ is true or false regardless of the exact timestamp values. Consequently, we can replace an existing timestamp $t'$ that became stable with another one that is always less than $<$ any other timestamp $t$ to be delivered. (Notice that $t$ cannot be replaced now until it also becomes stable, otherwise the concurrent semantics will break.) For instance, a stable timestamp $t'$ can be replaced by $\perp$, i.e., the least element (a.k.a., bottom) of the timestamp domain, effectively discarding the timestamp; e.g., 0 is the $\perp$ of a scalar timestamp in $\mathbb{N}$.

However, to perform such a timestamp replacement, two necessary conditions have to be met in the data-type-specific functions: queries should not be affected and the causal redundancy relations $\mathsf{R}$, $\mathsf{R}_0$, and $\mathsf{R}_1$ must not compare the existing PO-Log timestamps together. It was a fortunate happenstance that this was the case for all data types that we have considered in this paper, and thus all can be causally compacted using stabilization. The explanation of this is that, in all these data types, after causal redundancy is applied, the resulting simplified query operations no longer need to compare timestamps, and therefore, the queries will give the same result as before even if stable timestamps are replaced by $\perp$. On the other hand, $\mathsf{R}$, $\mathsf{R}_0$ and $\mathsf{R}_1$ never compare existing timestamps in the PO-Log with each other, but only compare them with the newly arrived operations.

This optimization can have substantial gains in many data types as it greatly diminishes the size of a replica state. In practice, instead of having timestamps that are maps or vectors with linear size on the number of replicas, we can have a special marker denoting $\perp$ (e.g., a null pointer). Consequently, we designate two interesting data type classes. The first class is those CRDTs where values may take considerably less space than timestamps, like sets of integers, and thus, stripping the timestamp from an element can reduce the storage overhead several orders of magnitude. The second class is those CRDTs whose state size can grow significantly; in such CRDTs, the percentage of elements in the PO-Log that are not yet causally stable will be quite small, most of operations being already stabilized. Therefore, this optimization reduces the storage overhead of the element itself, and may impact most PO-Log elements, as they become causally stable.

*Further optimizations in practice:* In actual implementations, the PO-Log can be split in two components: one that simply stores the set of stable operations and the other stores the timestamped operations. For instance, instead of being a map $T \hookrightarrow O$, the state is split into

$$\Sigma = \mathcal{P}(O) \times (T \hookrightarrow O)$$

detaching into a plain set of all stable operations. Furthermore, for some data types (such as AWSet) where only one kind of operations, i.e., $\mathsf{add}$, is ever in the PO-Log, the stable set of operations can become a plain set of elements (without timestamps), e.g., the traditional sequential set data type, with possibly specialized implementations according to the domain (i.e., a bitmap for dense sets of integers). To keep the presentation clear and consistent, we leave such optimizations to actual implementations and we rather only consider using a unified PO-Log, containing both stable $(\perp, o')$ and unstable operations $(t', o')$.

### 7.2.2 Discarding stable operations

This mechanism allows to discard stable operations, not only timestamps, if they have no impact on the semantics of the data type. The idea is that, for some data types like RWSet (explained next), we noticed that some operations could not be considered redundant once delivered, but they become useless once other future operations become stable; and therefore, it may be possible to discard a set of operations at once.

To support both causal stabilization mechanisms, we extend the framework, presented in Figure 10, to include a data-type-specific $\mathsf{stabilize}$ function. This function takes a stable timestamp $t$ (fed by the TCSB middleware) and the full PO-Log $s$ as input, and returns a new PO-Log (i.e., a map), possibly discarding a set of operations at once. The $\mathsf{stable}$ handler, on the other hand, invokes $\mathsf{stabilize}$ and then strips the timestamp (if the operation has not been discarded by $\mathsf{stabilize}$), by replacing a $(t', o')$ pair that is present in the returned PO-Log by $(\perp, o')$. In the following section, we show how the AWSet and MVRegister, presented in Figures 8 and 9, as well as other several data types, can be optimized with causal stabilization.

## 8 Portfolio of Pure Op-based CRDTs

In this section, we provide a catalog of CRDT designs following the pure op-based framework introduced above. Our purpose it to make this framework and CRDTs easy to understand and implement by designer and developers. We try to cover examples of the most used data types in practice, e.g., in Dynamo [13], Riak [12], Cassandra [35], etc. In particular, we address two categories of data types: (1) Commutative CRDTs: Grow-only Counter (GCounter), Positive-Negative Counter (PNCounter), Grow-only Set (GSet), and Two-Phase Set (2PSet); and Non-commutative data types: Enable-Wins Flag (EWFlag), Disable-Wins Flag (DWFlag), Multi-Value Register (MVRegister), Add-Wins Set (AWSet), and Remove-Wins Set (RWSet). To limit redundancy, but still provide reference specifications, the GCounter and both flags are collected in appendix.

$$
\begin{aligned}
\Sigma = T \hookrightarrow O & \qquad \sigma_i^0 = \{\} \\
\mathsf{prepare}_i(o, s) &= o \\
\mathsf{effect}_i(o, t, s) &= \{(t, o) \mid (t, o) \not\mathsf{R}\; s\} \cup \{x \in s \mid x \not\mathsf{R}_0\; (t, o) \\
&\quad \wedge (t, o)\; \mathsf{R}\; s \vee x \not\mathsf{R}_1\; (t, o) \wedge (t, o) \not\mathsf{R}\; s\} \\
\mathsf{stable}_i(t, s) &= \mathsf{stabilize}_i(t, s)[(\bot, o)/(t, o)] \\
\mathsf{R}, \mathsf{R}_0, \mathsf{R}_1 &= [\text{datatype-specific redundancy relations}] \\
\mathsf{stabilize}_i(t, s) &= [\text{datatype-specific stabilization function}] \\
\mathsf{eval}_i(q, s) &= [\text{datatype-specific query function}]
\end{aligned}
$$

Figure 10: Pure CRDTs framework including PO-Log compaction: causal redundancy and stabilization.

In this catalog, our methodology is to define the data type and show how it can be used in practice. Then we describe its semantics in a generic way before presenting its pure op-based design. This analogy helps the designer/developer choose the desired semantics for his business logic as well as understand the design of the CRDT in the Pure framework. Finally, we discuss additional optimizations that are data type specific when useful. (Recall that the notation used in this section are described in Section 2).

## 8.1 Commutative Data Types

According to Section 4, a CRDT is said to be commutative if all its possible operations are commutative over the state. The design of commutative CRDTs is datatype-specific, both in the pure op-based as well as in classical op-based approaches. Indeed, it is possible to design them using the PO-Log framework to be generic across all data types, as we do for other non-commutative data types; however, the tradeoff is to induce an extra unneeded overhead. We opt not to use the PO-Log for such data types and thus provide more practical designs. All the commutative designs assume the existence of a Reliable Causal Broadcast (RCB) middleware. The interaction between the presented CRDTs and the RCB are discussed in Algorithm 1, Section 3.

### 8.1.1 Positive-Negative Counter (PNCounter)

A PNCounter is a counter that supports increment and decrement operations. Therefore, it is very useful to represent quantities that can be added or removed, like the number of available tickets in a system. Its state is usually implemented as $n \in \mathbb{Z}$ with two operations $\mathsf{inc}$ (+ operator) and $\mathsf{dec}$ (− operator); thus, its value can go negative. Since + and − are commutative in $\mathbb{Z}$, PNCounter is a commutative data type.

Figure 11 conveys the pure CRDT design of PNCounter. The state is now defined as integer $z \in \mathbb{Z}$ that is initialized to 0. $\mathsf{prepare}$ can be either $\mathsf{inc}$ or $\mathsf{dec}$; however, in both cases, no meta-data is needed (and thus it is pure). The $\mathsf{effect}$ function increments or decrements $z$ depending on $\mathsf{prepare}$'s output. Finally, $\mathsf{eval}$ invokes the read operation $\mathsf{value}$ and simply returns $z$.

$$
\begin{array}{rcl}
\Sigma = \mathbb{Z} & & \sigma_i^0 = 0 \\
\mathsf{prepare}_i(\mathsf{inc}, z) & = & \mathsf{inc} \\
\mathsf{prepare}_i(\mathsf{dec}, z) & = & \mathsf{dec} \\
\mathsf{effect}_i(\mathsf{inc}, z) & = & z + 1 \\
\mathsf{effect}_i(\mathsf{dec}, z) & = & z - 1 \\
\mathsf{eval}_i(\mathsf{value}, z) & = & z
\end{array}
$$

Figure 11: Pure Positive-Negative Counter CRDT (PNCounter).

#### 8.1.2 Grow-only Set (GSet)

A GSet is a set where only adding items is supported. It is usually used when removals are not needed, as in storing the IPs of clients visiting a web-site. The set comprises a collection of items, of some type, that are added via add operation (a union $\cup$). Since add is commutative over the set, GSet is a commutative datatype.

Figure 12a shows the pure CRDT design of GSet. The state is represented by $s \in \mathcal{P}(V)$ (a power set of $V$) and is originally empty. The function prepare can only be the add operation with its parameter $v$, but without extra meta-data (which makes it pure). The function effect adds $v$ to the set using the set union operator: $\cup$. Finally, the eval function invokes the read operations elems and size to return all the elements of the set or its cardinality, respectively.

#### 8.1.3 Two-Phase Set (2PSet)

The 2PSet is a set where items can be added and removed, but never added after being removed. It can be seen as a composition of two GSet, one for additions and another for removals. In practice, the 2PSet can be used to represent collections having items with unique ids. The allowed mutating operations in 2PSet are add and rmv. They are commutative since the presence of an item in the removals set prevents re-adding that element later; Both $[\mathsf{add}, a]$ followed by $[\mathsf{rmv}, a]$, and $[\mathsf{rmv}, a]$ followed by $[\mathsf{add}, a]$ will have the same effect, i.e., $a$ not being present. Therefore — contrary to what it looks like at a glance — a 2PSet is a commutative CRDT.

Figure 12b depicts the pure CRDT design of the 2PSet. The state is designed as two power sets $s, p \in \mathcal{P}(V)$, initially empty, where $s$ retains the added values and $p$ retains the removed ones. A prepare can either be add or rmv together with a parameter $v$ and without additional meta-data (i.e., making the CRDT pure). When item $v$ is added, the effect unions $v$ to set $s$ only of it is not in the set of removals $p$ (i.e., $s \cup (\{v\} \setminus p)$), which remains intact in this case. If $v$ is to be removed via rmv, it is subtracted from $s$ and unioned to $p$. Notice that, set $s$ can grow and shrink in size depending on the issued operations whereas $p$ can only grow in size as long as rmv operations are issued. This imposes a space overhead, but ensures that removed items are never added again. Finally, the eval function invokes the read operations elems and size to return all the elements in $s$ or its cardinality, respectively.

$$\Sigma = \mathcal{P}(V) \qquad \sigma_i^0 = \{\}$$

$$\begin{aligned}
\mathsf{prepare}_i([\mathsf{add}, v], s) &= [\mathsf{add}, v] \\
\mathsf{effect}_i([\mathsf{add}, v], s) &= s \cup \{v\} \\
\mathsf{eval}_i(\mathsf{elems}, s) &= s \\
\mathsf{eval}_i(\mathsf{size}, s) &= |s|
\end{aligned}$$

(a) GSet

$$\Sigma = \mathcal{P}(V) \times \mathcal{P}(V) \qquad \sigma_i^0 = (\{\}, \{\})$$

$$\begin{aligned}
\mathsf{prepare}_i([\mathsf{add}, v], \sigma) &= [\mathsf{add}, v] \\
\mathsf{prepare}_i([\mathsf{rmv}, v], \sigma) &= [\mathsf{rmv}, v] \\
\mathsf{effect}_i([\mathsf{add}, v], (s, p)) &= (s \cup (\{v\} \setminus p), p) \\
\mathsf{effect}_i([\mathsf{rmv}, v], (s, p)) &= (s \setminus \{v\}, p \cup \{v\}) \\
\mathsf{eval}_i(\mathsf{elems}, (s, p)) &= s \\
\mathsf{eval}_i(\mathsf{size}, (s, p)) &= |s|
\end{aligned}$$

(b) 2PSet

Figure 12: Pure Grow-only Set and Two-Phase Set CRDTs.

## 8.2 Non-Commutative Data Types

These CRDTs accept operations that are not commutative. The challenge is then to design the effect function to be commutative. The design of such data types follows the framework presented in Figure 10 and supports using the PO-Log as well as the compaction mechanisms explained in the preceding sections. Consequently, the presented pure CRDTs have a common design for the state $\Sigma$, prepare, effect, and stable functions. Although the other functions like R, stabilize, and eval are datatype-specific, they are often fairly simple as shown next. As explained earlier, non-commutative pure CRDTs assume the existence of an Tagged Causal Stable Broadcast middleware (TCSB), as presented in Section 5. The interplay between the TCSB and the pure CRDTs is described in Algorithm 2.

Since the semantics of non-commutative CRDTs is more complex than commutative ones, being concurrent, we formally define their concurrent semantics assuming that all operations are retained in a PO-Log, where the output of the query operations defines its semantics. This makes the definition very solid and stands as a base-line to verify future amendments across; i.e., the output of defined queries for a datatype must not change in any new design or optimization (which is true in all the designs we present).

### 8.2.1 Multi-Value Register (MVRegister)

Contrary to a classical register that holds a single value at a time, a MVRegister can hold multiple values if their corresponding write operations are concurrent; thus letting the application choose the desired values returned. One motivation for MVRegister was to mimic the design of the Amazon Shopping Cart in Dynamo DB [13]. Since the order of two write operations on the MVRegister can lead to different results if one of them is in the causal future of the other, the datatype is non commutative.

The concurrent semantics of MVRegister is depicted in Figure 13 and says that a read rd should return all write wr operations that are not causally succeeded by any other operation (i.e., neither wr nor clear). Consequently, the first clause in the Pure CRDT design, in Figure 14, says that clear operations must not be added to the PO-Log. The second clause then means that wr operations are removed from the PO-Log if there is any wr or clear operation in the causal future; otherwise, concurrent wr operations will be kept. The rd operation, therefore, simply returns all available wr operations in the PO-Log. Finally,

$$\mathsf{eval}_i(\mathsf{rd}, s) \quad = \quad \{v \mid (t, [\mathsf{wr}, v]) \in s \wedge \forall (t', \_) \in s \cdot t \not< t'\}$$

Figure 13: MVRegister concurrent semantics

$$\begin{aligned}
(t, o) \ \mathsf{R} \ s &\iff o[0] = \mathsf{clear} \\
(t', o') \ \mathsf{R_} \ (t, o) &\iff t' < t \\
\mathsf{stabilize}_i(t, s) &= s \\
\mathsf{eval}_i(\mathsf{rd}, s) &= \{v \mid (_, [\mathsf{wr}, v]) \in s\}
\end{aligned}$$

Figure 14: Compact MVRegister CRDT

stabilize has no effect on the PO-Log, and thus the calling function stable will simply replace causally stable timestamps with $\perp$.

### 8.2.2 Sets (AWSet and RWSet)

A set is a very well known data structure that retains unordered items, being added and removed through add and rmv operations, respectively. Obviously, these operations are not commutative and thus reordering them will yield different effects, making the datatype non commutative. Special consideration must be given when these operations are concurrent. Typically, the designers explicitly define extra rules to make the behavior consistent across different replicas, thus leading to two variants: Add-Wins Set (AWSet) and Remove-Wins Set (RWSet).

In AWSet, an add operation dominates a concurrent rmv that is mutating the same item in the set. This is explained formally in the concurrent semantics in Figure 15: a elems operation will return all the values associated with add such that no corresponding rmv operation or clear operation are found in the causal future of the add. Consequently, the first clause in the Pure CRDT design in Figure 16 specifies that newly delivered rmv and clear operations are never added to the PO-Log; which leaves the PO-Log with only add operations. The second clause however says that an existing add in the PO-Log is considered redundant if there is another add or rmv operations on the same item (specified by $o[1]$ in Figure 16) or a clear operation such that they are in the causal future of the candidate add. Therefore, the elems operation will simply return the items associated with an add in the PO-Log, since neither duplicate adds nor rmv and clear operations exist in the PO-Log. For the same reasons as previous data types, stabilize has no effect and the calling stable function will only replace the causally stable timestamps with $\perp$.

On the other hand, in RWSet, a rmv operation dominates a concurrent add that is mutating the same item in the set, thus masking its effect. Therefore, the formal concurrent semantics in Figure 17 says that a elems operation on a PO-Log returns all values associated with an add such that all rmv operations are in its causal past, and clear operations are not in the causal future. Notice that, contrary to AWSet, concurrent add operations with a rmv will be discarded (reflecting the remove-wins semantics). The corresponding Pure CRDT design of RWSet is presented in Figure 18. In particular, the first clause prevents any clear operation from being added to PO-Log. The second clause specifies that a candidate

$$
\begin{aligned}
\mathsf{eval}_i(\mathsf{elems}, s) \quad &= \quad \{v \mid (t, [\mathsf{add}, v]) \in s \\
&\qquad \wedge \forall (t', [\mathsf{rmv}, v] \vee \mathsf{clear}) \in s \cdot t \not< t'\}
\end{aligned}
$$

Figure 15: AWSet concurrent semantics

$$
\begin{aligned}
(t, o)\ \mathsf{R}\ s \quad &= \quad o[0] = (\mathsf{clear} \vee \mathsf{rmv}) \\
(t', o')\ \mathsf{R\_}\ (t, o) \quad &= \quad t' < t \wedge (o[0] = \mathsf{clear} \\
&\qquad \vee\ o[1] = o'[1]) \\
\mathsf{stabilize}_i(t, s) \quad &= \quad s \\
\mathsf{eval}_i(\mathsf{elems}, s) \quad &= \quad \{v \mid (\_, [\mathsf{add}, v]) \in s\}
\end{aligned}
$$

Figure 16: Compact AWSet Pure CRDT

operation on item $v$ is only removed if there is another operation on $v$ or a $\mathsf{clear}$ in its causal future. This actually means that $\mathsf{rmv}$ operations can still live in the PO-Log, to the contrary of AWSet. In fact, $\mathsf{rmv}$ cannot be removed since a concurrent $\mathsf{add}$ on the same item may be delivered which breaks the remove-wins semantics. Even more, an $\mathsf{add}$ operation $a$ cannot be removed despite the presence of concurrent $\mathsf{rmv}$ operation $r$ (as one might think). The reason is that $r$ can become stable at any time; if it happened to be followed by a $\mathsf{clear}$ operation, i.e., in its causal future, all stable operations in the PO-Log will be deleted, $r$ among them. Since $a$ is not yet stable, and thus concurrent with $\mathsf{clear}$, the former will stay in the PO-Log. This breaks the remove-wins semantics since $r$ and $a$ were originally concurrent and hence $r$ should win. These reasons lead to a more complex $\mathsf{stabilize}$ function (explained next) that is in charge of removing the extra operations that were not safe to be removed previously through $\mathsf{R}$ or $\mathsf{R\_}$.

We explain the $\mathsf{stabilize}$ function, depicted in Figure 18, with the help of a simulation of the PO-Log state transitions upon stabilization in Figure 19. The $\mathsf{stabilize}$ function tries to remove redundant operations once invoked by $\mathsf{stable}$ when a timestamp $t$ becomes stable. The removed operations are defined in three clauses, i.e., those removed from the PO-Log using the *set minus* operator in Figure 18. The first clause specifies that, once $t$ becomes stable, the associated $\mathsf{add}$ operations are removed from the PO-Log as long as there is at least another operation in the PO-Log with a different timestamp $t' \neq t$ (otherwise the latter operation will also be stable). This clause corresponds to the first four cases in Figure 19: once an $\mathsf{add}$ operation $a$ becomes stable, the existence of other $\mathsf{add}$s will make it redundant, as they add the same item to the set; $a$ is also removed if there exist any other $\mathsf{rmv}$ operation $r$ on the same item since, according to the remove-wins semantics, $r$ wins the concurrent $a$ which has just became stable, and thus, it is not expected to have another concurrent $\mathsf{add}$ delivered (by definition of causal stability). The second clause in $\mathsf{stabilize}$ (Figure 18) deletes the stable $\mathsf{rmv}$ operations as long as the potential non stable operations on the same item are not uniquely $\mathsf{add}$ operations. This is explained in the simulation in Figure 19 in cases (5) through (8): cases (7) and (8) are trivial since a $\mathsf{rmv}$ is simply redundant by duplicate $\mathsf{rmv}$s; case (5) removes a stable $\mathsf{rmv}$ if no other operation exist. The rational is that the stable $\mathsf{rmv}$ being stable, no concurrent operations are expected, meaning that any

$$\begin{aligned}\mathsf{eval}_i(\mathsf{elems}, s) \quad = \quad & \{v \mid (t, [\mathsf{add}, v]) \in s \\ & \wedge \forall (t', [\mathsf{rmv}, v]) \in s \cdot t' < t \\ & \wedge \forall (t'', \mathsf{clear}) \in s \cdot t \not< t''\}\end{aligned}$$

Figure 17: RWSet concurrent semantics

$$\begin{aligned}(t, o) \ \mathsf{R} \ s \quad &= \quad o[0] = \mathsf{clear} \\ (t', o') \ \mathsf{R}\_ \ (t, o) \quad &= \quad t' < t \wedge (o[0] = \mathsf{clear} \vee o[1] = o'[1]) \\ \mathsf{stabilize}_i(t, s) \quad &= \quad s \setminus \\ & \quad\ \{(t, [\mathsf{add}, e]) \in s \mid \exists (t', [\_, e]) \in s \cdot t \neq t'\} \setminus \\ & \quad\ \{(t, [\mathsf{rmv}, e]) \in s \mid \\ & \quad\ \ \{op \mid (t', [op, e]) \in s \wedge t' \neq t\} \neq \{\mathsf{add}\}\} \setminus \\ & \quad\ \{(\perp, [\mathsf{rmv}, e]) \in s \mid \forall (t', [\mathsf{add}, e]) \in s \cdot t' = t\} \\ \mathsf{eval}_i(\mathsf{elems}, s) \quad &= \quad \{v \mid (\_, [\mathsf{add}, v]) \in s \wedge (\_, [\mathsf{rmv}, v]) \notin s\}\end{aligned}$$

Figure 18: Compact Pure RWSet CRDT

delivered operations will be in its causal future, and thus, making stable rmv useless. Case (6) does not lead to any removals. Indeed, if the stable rmv operation $r$ is removed, any existing add operation on the same item $a$ will win, which contradicts with the remove-wins semantics since $r$ and $a$ are originally concurrent. This practically means the stable rmv will be kept in the PO-Log for a later stage, thus forking the cases (9) and (10). When an add operation becomes stable, i.e., all add and rmv operations on the same item are not stable, they can both be removed from the PO-Log since rmv wins, and it is not expected to have any concurrent operation delivered any more. To the contrary, if there is at least another add operation, the stable add will be removed while the stable rmv will be kept for similar reasons as in point (6) explained above.

After invoking stabilize and removing all redundant operations, the stable function will replace the stable timestamps with $\perp$. Finally, the elems operation will return all the items that are associated with add if there is no corresponding rmv in the PO-Log.

# 9 Discussion

We recap this work by discussing the most substantial benefits the Pure CRDT model brings on the conceptual and performance levels. We divide our discussion into the following three subsections.

## 9.1 Making Op-based CRDTs Op-based

The first incentive behind this work was to remove the confusion between op-based and state-based CRDTs [32, 1]. In fact, the nomenclature of these two approaches refers to the type of the message being disseminated: the message is generally (supposed to be) the "operation"

| | | | |
|---|---|---|---|
| (1) | $\dot{\boxed{\text{a}}}$ | $\rightarrow$ | $\dot{\boxed{\text{a}}}$ |
| (2) | $\dot{\boxed{\text{a}}}a^+$ | $\rightarrow$ | $\cancel{\dot{\boxed{\text{a}}}}a^+$ |
| (3) | $\dot{\boxed{\text{a}}}r^+$ | $\rightarrow$ | $\cancel{\dot{\boxed{\text{a}}}}r^+$ |
| (4) | $\dot{\boxed{\text{a}}}a^+r^+$ | $\rightarrow$ | $\cancel{\dot{\boxed{\text{a}}}}a^+r^+$ |
| (5) | $\dot{\boxed{\text{r}}}$ | $\rightarrow$ | $\cancel{\dot{\boxed{\text{r}}}}$ |
| (6) | $\dot{\boxed{\text{r}}}a^+$ | $\rightarrow$ | $\dot{\boxed{\text{r}}}a^+$ |
| (7) | $\dot{\boxed{\text{r}}}r^+$ | $\rightarrow$ | $\cancel{\dot{\boxed{\text{r}}}}r^+$ |
| (8) | $\dot{\boxed{\text{r}}}a^+r^+$ | $\rightarrow$ | $\cancel{\dot{\boxed{\text{r}}}}a^+r^+$ |
| (9) | $\boxed{\text{r}}\ \dot{\boxed{\text{a}}}$ | $\rightarrow$ | $\cancel{\boxed{\text{r}}}\cancel{\dot{\boxed{\text{a}}}}$ |
| (10) | $\boxed{\text{r}}\ \dot{\boxed{\text{a}}}a^+$ | $\rightarrow$ | $\boxed{\text{r}}\cancel{\dot{\boxed{\text{a}}}}a^+$ |

Figure 19: Simulation of the possible transitions at stabilization in RWSet. The operations add and rmv, i.e., $r$ and $a$ respectively, are assumed to mutate the same item. A boxed operation is stable. A dotted operation refers to the operation that has just became stable. The sign $+$ is used to indicate the presence of duplicate operations. The canceled boxed operations on the right side are the redundant operations due to causal stabilization.

in the operation-based model whereas a "state" in the state-based model. This notion is, however, not reflected in the classical op-based design since the disseminated message, i.e., the output of prepare, comprises other meta-data as we have seen in Figure 2b, Section 3. In many cases, this leads to other issues (discussed next) like ad-hoc designs and abuse of resources. The Pure op-based CRDTs we presented are "pure" op-based designs that only require the operation (and potential parameters) to be disseminated; thus bringing back the essence of the "op-based" terminology. The tangible benefits of this conceptual feature is reflected in the following sections.

## 9.2 Making Op-based CRDT Designs (Almost) Generic

The Pure CRDT framework makes the design of op-based CRDTs "almost" generic. In our experience, making them fully generic is impossible due to the native semantic discrepancy of the designed data types — making them more generic will be impractical as we've seen in the PO-Log based CRDTs before compaction (presented in Section 4).

The classical op-based CRDT designs are data type dependent. Observing the seminal CRDT designs in [32] shows that designing a new CRDT will require the designer/developer to completely define every design component, i.e., $\Sigma$, prepare for each operation type, effect for each operation type, and eval. In the Pure CRDT framework we introduced, the basic components are generic: $\Sigma$, prepare, effect, and stable. Although our approach requires additional data-type-specific components like R, R_, and stabilize, we argue that their definitions are very simple; and as you may observe in the catalog of CRDTs in the previous section, these mere components can be implemented in a couple of conditionals or assertions. Nevertheless, our recommendation was not to use the PO-Log for commutative CRDTs, being simple and more efficient, even if it will be generic.

Finally, introducing a generic framework for CRDTs brings notable benefits. First, it reduces the complexity of the design and allows the designer/developer to focus on the semantics of a data type rather than fiddle with other design details. Second, it reduces the implementation cost since the common components can now be implemented once and used by all data types, avoiding repetitions. Third, it reduces the likelihood of introducing new bugs and anomalies as the design limits the freedom of the developer to change the rigorously thought-out generic components.

### 9.3 Making Op-based CRDT Efficient

Our Pure CRDT model improves the efficiency of dissemination and storage. In classical op-based CRDT designs, additional meta-data is usually used to express the causality information required by non commutative data types [32]. An example we've seen in this paper is the AWSet, in Figure 2b, where the state $\Sigma = \mathbb{N} \times \mathcal{P}(I \times \mathbb{N} \times V)$ comprises a set of triples, and the output of prepare for the remove operation (which must be broadcast to other replicas) contains also a set of triples. On the dissemination front, we believe that there is abuse of network resources since the causality meta-data (mainly timestamps) is already being exchanged, though not used, in the underlying middleware. The introduction of Tagged Causal Stable Broadcast middleware (TCSB) exploits this information which allows designing pure CRDTs, i.e., where the prepare output (to be disseminated) is only the operation name and potential arguments. The dissemination overhead can be reduced several orders of magnitude when the fraction of data to timestamps is small, e.g., when the data is an integer whereas the timestamp is a vector clock. TCSB has also other advantages on the storage front through the causal stability information that helps pruning the useless meta-data in the PO-Log, which as we've seen, turns the CRDT into a classical sequential data type in some cases. As explained in Section 7, the gain will be higher in data types that exhibit large storage sizes (e.g, as set of millions of items) since most of the operations will already be causally stable and thus compacted. Measuring the real overhead across real data patterns is subject future work.

## 10 Related Work

*Causal broadcast* The early definitions of *causality* were introduced by Lamport in [20]. Schiper et al. introduced an algorithm in [29] to implement *causal ordering* based on message delivery. The authors distinguished it from Lamports causality definitions which they called *causal timestamping*. Then, causal broadcast has been explored in the context of process group communication in [8] to ensure delivery orders that do not contradict potential causality [20]. Implementations can piggyback to messages their causally preceding messages or tag it with a vector clock [30, 23, 14] and ensure that preceding messages are delivered first.

*Weakly Consistent Replication.* The design of replicated systems that are always available and eventually converge can be traced back to historical designs in [37, 18, 26], among others. Lazy Replication [19] allows enforcing causal consistency, but may apply concurrent operations in different orders in different replicas, possibly leading to divergence if operations are not commutative; TSAE [16] also either applies concurrent operations in possibly different orders, or allows enforcing a total order compatible with causality, at the cost of delaying message delivery. Both these systems use a message log, the former with complete causality information, but the log is *pre-delivery*, unseen by the application: operations are applied sequentially to the current state and queries use only the state. In our framework the PO-Log is *post-delivery*, being part of the data type state, maintains causality information and is used in query operations.

*Conflict-Free Replicated Data Types.* The formalization of the commutativity requirements for concurrent operations in replicated data types was introduced in [21, 27], and that of the state based semi-lattices was presented in [4]. Afterwards, the integration of the two models with many extensions was presented in Conflict-free Replicated Datatypes [31, 33]. Currently, CRDTs have made their way into the industry through designing highly available scalable systems in cloud databases like RIAK [11], and mobile gaming industry such as Rovio [28].

*Message Stability.* The notion of message stability was defined in [7] to represent a message that has been received by all recipients; each replica can discard any message it knows to be stable after delivering it. Similar notions are used in Lazy Replication [19] and TSAE [16]. In all these cases the aim is message garbage collection. Our definition of *causal stability* is the stronger notion that *no more concurrent messages will be delivered*; we use it inside the Datatypes to discard causality information while keeping the operation. Causal stability is close to what is used in the mechanics of Replicated Growable Arrays (RGA) [27], although no definition is presented there.

*Message Obsolescence.* Semantically reliable multicast [25] uses the concept of *message obsolescence* to avoid delivering messages made redundant by some newly arrived message, where obsolescence is a strict partial order that is a subset of causality, possibly relating messages from the same sender or totally ordered messages from different senders. Our obsolescence relation is more general, being defined on clock-operation pairs, and can relate concurrent messages. Also, it is defined per-data-type, being used inside each data type, post-delivery.

# 11 Conclusions

We introduced Pure operation-based CRDTs: a novel generic and efficient op-based CRDT model that establishes a clear frontier with state-based models. The model is "pure" in the sense that a disseminated message only contains the operation name and its potential arguments. The causal information that is usually explicitly retained in the CRDT state and added to disseminated messages, in the classical op-based CRDT, are now provided by a Tagged Causal Stable Broadcast (TCSB) middleware that we proposed. Despite the useful design and performance benefits it brings to CRDTs, we tried to introduce the TCSB in a self-contained section as we believe it is an important extension that can be exploited by the community to improve other concurrent designs as well. We also believe that the TCSB deserves a dedicated practical analysis in the future to understand its characteristics and performance tradeoffs.

Our paper distinguishes between two types of CRDTs: commutative and non commutative CRDTs. For the non commutative CRDTs, we design the state as a partially ordered log (PO-Log) that can retain concurrent operations tagged with timestamps. The timestamps and some operations can be discarded using causal redundancy and stabilization that are features supported by the TCSB. On the other hand, the commutative CRDTs are not based on PO-Log being very simple. Indeed, they can be designed using the PO-Log if desired, we however don't recommend this due to the needless overhead and complexity of the PO-Log in this case.

We tried to provide a catalog of Pure CRDTs addressing many useful data types in practice. We believe that these designs are now simpler, provided that the generic framework is well understood. Some data types, like Remove-Wins Sets are however more complex due to their non-trivial concurrent semantics especially when used with clear operations that reset the state. We noticed that the framework can be easier if reset operations are not needed. We did not address more composite data types like Maps since this will require extending the PO-Log, probably to a Multi-PO-Log, thus imposing more complexity to the paper, which we opt to avoid. Finally, we believe that a detailed empirical analysis that studies the different data access patterns to the CRDTs in different models will help better understand the performance tradeoffs; we plan to do such analysis in the future.

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# Additional data types

## Commutative Data Types

### Grow-only Counter (GCounter)

As its name indicates, a GCounter is a counter that can only be incremented. Among its use-cases are implementing vector clocks in distributed systems or counting the number of clients visiting a web-site. In GCounter, the state is usually an integer $n \in \mathbb{N}$ and the only mutating operation is $\mathsf{inc}$, which reads the current state and increments it by 1. Since $+$ is commutative over $\mathbb{N}$, GCounter is a commutative data type.

Figure 20 depicts a pure CRDT design of GCounter. The design is self explanatory: the state is an integer $n \in \mathbb{N}$ initialized to 0; $\mathsf{prepare}$ can only be $\mathsf{inc}$ without any meta-data (and thus the design is pure). The $\mathsf{effect}$ function simply increments $n$ by 1, whereas $\mathsf{eval}$ invokes the read operation $\mathsf{rd}$ which returns $n$.

## Non-Commutative Data Types

### Flags (EWFlag and DWFlag)

A flag is a datatype that retains a single boolean value indicating whether it is "enabled" or "disabled". Flags [12] have several uses in practice, e.g., showing if an email has been read. Since the two allowed mutating operations, i.e., $\mathsf{enable}$ and $\mathsf{disable}$, can be concurrent and result in opposite outputs depending on the delivery order, a flag is obviously a non commutative CRDT. To ensure a consistent result on all replicas when $\mathsf{enable}$ and $\mathsf{disable}$ operations are concurrent, a design decision is usually made to break this tie, thus yielding two flag versions: Enable-Wins Flag (EWFlag) where two concurrent $\mathsf{enable}$ and $\mathsf{disable}$ operations leave the flag "enabled", to the contrary of a Disable-Wins Flag (DWFlag). The formal concurrent semantics of EWFlag and DWFlag as well as their pure CRDT designs are depicted in Figures 21, 22, and 23, 24, respectively.

The EWFlag semantics (in Figure 21) says that a $\mathsf{read}$ operation on the PO-Log must return "enabled" iff there is an $\mathsf{enable}$ operation that is not strictly followed by a $\mathsf{disable}$ or $\mathsf{clear}$ (i.e., a reset) operation in its causal future; meaning that, a concurrent $\mathsf{disable}$ or $\mathsf{clear}$ will not "disable" the flag. Consequently, the first clause in the pure design in Figure 22 says that a newly delivered operation is always considered redundant, and never added to the PO-Log, if it is $\mathsf{disable}$ or $\mathsf{clear}$. The second clause, however, compares the existing operations in the PO-Log to remove any operation $o$ that is strictly followed by another operation $o'$ in its causal future, i.e., if $t < t'$. Since only $\mathsf{enable}$ operations are retained by the PO-Log (as the first clause specifies), the second clause will actually remove any

$$\begin{array}{rcl}
\Sigma = \mathbb{N} & & \sigma_i^0 = 0 \\
\mathsf{prepare}_i(\mathsf{inc}, n) & = & \mathsf{inc} \\
\mathsf{effect}_i(\mathsf{inc}, n) & = & n + 1 \\
\mathsf{eval}_i(\mathsf{value}, n) & = & n
\end{array}$$

Figure 20: Pure Grow-only Counter CRDT (GCounter).

$$\begin{aligned}\mathsf{eval}_i(\mathsf{read}, s) &= \exists (t, \mathsf{enable}) \in s \cdot \forall (t', o') \in s \cdot \\ & \quad (o' = \mathsf{disable} \vee \mathsf{clear}) \cdot t \not< t'\end{aligned}$$

Figure 21: EWFlag concurrent semantics

$$\begin{aligned}(t, o)\ \mathsf{R}\ s &= o[0] = (\mathsf{disable} \vee \mathsf{clear}) \\ (t', o')\ \mathsf{R}\_\ (t, o) &= t' < t \\ \mathsf{stabilize}_i(t, s) &= s \\ \mathsf{eval}_i(\mathsf{read}, s) &= (\_, \mathsf{enable}) \in s\end{aligned}$$

Figure 22: Compact EWFlag Pure CRDT

enable operations in the PO-Log with the $t < t'$ condition. Consequently, the output of the read operation, invoked by eval, returns an existing enable operation, or empty otherwise, yielding the same output of the read in the base-line semantics in Figure 21. Notice that stabilize has no effect in this datatype, and thus always returns the PO-Log as is; which means that stable only replaces the stable timestamps in the PO-Log with $\perp$.

$$\begin{aligned}\mathsf{eval}_i(\mathsf{read}, s) &= \exists (t, \mathsf{enable}) \in s \cdot \forall (t', \mathsf{disable}) \in s \cdot t' < t \\ & \quad \wedge \forall (t'', \mathsf{clear}) \in s \cdot t \not< t''\end{aligned}$$

Figure 23: DWFlag concurrent semantics

$$\begin{aligned}(t, o)\ \mathsf{R}\ s &= o[0] = \mathsf{clear} \\ (t', o')\ \mathsf{R}\_\ (t, o) &= t' < t \\ \mathsf{stabilize}_i(t, s) &= s \\ \mathsf{eval}_i(\mathsf{read}, s) &= (\_, \mathsf{enable}) \in s \wedge (\_, \mathsf{disable}) \notin s\end{aligned}$$

Figure 24: Compact DWFlag Pure CRDT

As for DWFlag, the concurrent specs (in Figure 23) says that the flag is enabled as long as there is at least one enable operation where all existing disable operations are in its strict causal past, and all existing clear operations are either concurrent or in its causal past. Consequently, the first clause in the pure CRDT design, in Figure 24, only removes newly delivered clear operations. The second clause however specifies that disable and enable operations are only removed from the PO-Log if they are succeeded by another operation with a greater timestamp. To reflect this behavior, the read operation returns the enable operations such that no disable operations exist in the PO-Log (in which case a disable should prevail), yielding an equivalent output to the base-line read operation in Figure 23. Finally, stabilize has no effect in this datatype as well.