On Weighted Configuration Logics

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Abstract. We introduce and investigate a weighted propositional configuration logic over a commutative semiring. Our logic, which is proved to be sound and complete, is intended to serve as a specification language for software architectures with quantitative features. We extend the weighted configuration logic to its first-order level and succeed in describing architecture styles equipped with quantitative characteristics. We provide interesting examples of weighted architecture styles. Surprisingly, we can construct a formula, in our logic, which describes a classical problem of a different nature than that of software architectures.

Keywords: Software architectures, configuration logics, semirings, weighted configuration logics

1 Introduction

Architecture is a critical issue in design and development of complex software systems. Whenever the construction of a software system is based on a "good" architecture, then the system satisfies most of its functional and quality requirements. But what are the characteristics of a "good" architecture and how one can design it? Despite the huge progress on software architecture, over almost three decades, the field remains relatively immature (cf. [5] for an excellent presentation of the progress of software architecture). Several fundamental matters still remain open, for instance the distinction between architectures and their properties. Recently in [11], the relation among architectures and architecture styles has been studied. An architecture style describes a family of "similar" architectures, i.e., architectures with the same types of components and topologies. The authors introduced the propositional configuration logic (*PCL* for short) which was proved sufficient to describe architectures: the meaning of every *PCL* formula is a configuration set, and every architecture can be represented by a configuration on the set of its components. The first-order and second-order extensions of *PCL* described perfectly the concept of architecture styles. Therefore, *PCL* and its first- and second-order extensions constitute logics for the specification of architecture styles and hence, an important contribution to rigorous systems design (cf. [14]).

In this paper we introduce and investigate a weighted *PCL* over a commutative semiring $(K, \oplus, \otimes, 1, 0)$. Our work is motivated as follows. *PCL* and its first- and second-order extensions of [11] describe qualitative features of architectures and architecture styles. Weighted *PCL* describes quantitative features of architectures, and weighted first-order configuration logic describes quantitative features of architecture styles. For instance, the costs of the interactions among the components of an architecture, the time needed, the probability of the implementation of a concrete interaction, etc. Our weighted *PCL* consists of the *PCL* of [11] which is interpreted in the same way, and a copy of it which is interpreted quantitatively. This formulation has the advantage that practitioners can use the *PCL* exactly as they are used to, and the copy of it for the quantitative interpretation. The semantics of weighted *PCL* formulas are polynomials with values in the semiring K. The semantics of (unweighted) *PCL* formulas take only the values 1 and 0 corresponding to *true* and *false*, respectively. Weighted logics have been considered so far in other set-ups. More precisely, the weighted *MSO* logic over words, trees, pictures, nested words, timed words, and graphs (cf. [1]), the weighted *FO* logic [8–10], the weighted *LTL* (cf. for instance [3] and the references in that paper), the weighted *LDL* [3], as well as the weighted *MSO* logic and *LDL* over infinite alphabets [13], and the weighted μ-calculus and CTL [7].

The main contributions of our work are the following. We prove that for every weighted *PCL* formula we can effectively construct an equivalent one in full normal form which is unique up to the equivalence relation. Furthermore, our weighted *PCL* is sound and complete. Both the aforementioned results hold also for *PCL* and this shows the robustness of the theory of *PCL*. We prove several properties for the weighted first-order configuration logic and in addition for its Boolean counterpart of [11]. We present as an example the weighted *PCL* formula describing the Master/Slave architecture with quantitative features. According to the underlying semiring, we get information for the cost, probability, time, etc. of the implementation of an interaction between a Master and a Slave. We construct a weighted first-order configuration logic formula for the Publish/Subscribe architecture style with additional quantitative characteristics. Surprisingly, though *PCL* was mainly developed as a specification language for architectures, we could construct a weighted *PCL* formula describing the well-known travelling salesman problem.

Apart from this introduction the paper contains 5 sections. In Section 2 we present preliminary background needed in the sequel. In Section 3 we introduce the weighted proposition interaction logic which describes quantitative interactions among the components of an architecture. Then, in Section 4 we introduce the weighted *PCL* and investigate the main properties of the semantics of weighted *PCL* formulas. Section 5 is devoted to the construction of the unique full normal form (modulo the equivalence relation) equivalent to a given weighted *PCL* formula. Furthermore, it contains the results for the soundness and completeness of the weighted *PCL*. In Section 6, we extend the weighted *PCL* to its first-order level. We prove several properties for weighted first-order configuration logic formulas as well as for first-order configuration logic formulas of [11]. Finally, in the conclusion, we list several open problems for future research. Due to space limitations we skip detailed proofs of our results. We refer the interested reader to the full version of our paper on arXiv [12].

2 Preliminaries

A *semiring* $(K, \oplus, \otimes, 0, 1)$ consists of a set K, two binary operations $\oplus$ and $\otimes$ and two constant elements 0 and 1 such that $(K, \oplus, 0)$ is a commutative monoid, $(K, \otimes, 1)$ is a monoid, multiplication distributes over addition, and $0 \otimes k = k \otimes 0 = 0$ for every $k \in K$. If the monoid $(K, \otimes, 1)$ is commutative, then the semiring is called commutative. The semiring is denoted simply by K if the operations and the constant elements are understood. The result of the empty product as usual equals to 1. The semiring K is called (additively) idempotent if $k \oplus k = k$ for every $k \in K$. The following algebraic structures are well-known semirings: the semiring $(\mathbb{N}, +, \cdot, 0, 1)$ of natural numbers, the Boolean semiring $B = (\{0, 1\}, +, \cdot, 0, 1)$, the tropical or min-plus semiring $\mathbb{R}_{\min} = (\mathbb{R}_+ \cup \{\infty\}, \min, +, \infty, 0)$ where $\mathbb{R}_+ = \{r \in \mathbb{R} \mid r \geq 0\}$, the arctical or max-plus semiring $\mathbb{R}_{\max} = (\mathbb{R}_+ \cup \{-\infty\}, \max, +, -\infty, 0)$, the Viterbi semiring $([0, 1], \max, \cdot, 0, 1)$ used in probability theory, and every bounded distributive lattice with the operations sup and inf, especially the fuzzy semiring $F = ([0, 1], \max, \min, 0, 1)$. Trivially all the previous semirings are commutative, and all but the first one are idempotent.

Let Q be a set. A *formal series* (or simply *series*) *over* Q *and* K is a mapping $s : Q \to K$. The *support of* s is the set $\mathrm{supp}(s) = \{q \in Q \mid s(q) \neq 0\}$. A series with finite support is called also a *polynomial*. We denote by $K \langle\langle Q \rangle\rangle$ the class of all series over Q and K, and by $K \langle Q \rangle$ the class of all polynomials over Q and K. Let $s, r \in K \langle\langle Q \rangle\rangle$ and $k \in K$. The *sum* $s \oplus r$, the *products with scalars* ks and sk, and the *Hadamard product* $s \otimes r$ are defined elementwise, respectively by $s \oplus r(v) = s(v) \oplus r(v)$, $ks(v) = k \otimes s(v)$, $sk(v) = s(v) \otimes k$, $s \otimes r(v) = s(v) \otimes r(v)$ for every $v \in Q$. Trivially, if the series s, r are polynomials, then the series $s \oplus r, ks, sk, s \otimes r$ are also polynomials.

Throughout the paper K will denote a commutative semiring.

3 Weighted propositional interaction logic

In this section, we introduce the weighted propositional interaction logic. For this, we need to recall first the propositional interaction logic [11].

Let P be a nonempty finite set of *ports*. We let $I(P) = \mathcal{P}(P) \setminus \{\emptyset\}$, where $\mathcal{P}(P)$ denotes the power set of P. Every set $a \in I(P)$ is called an *interaction*. The syntax of *propositional interaction logic* (*PIL* for short) formulas over P is given by the grammar

$$\phi ::= true \mid p \mid \overline{\phi} \mid \phi \vee \phi$$

where $p \in P$. As usual, we set $\overline{\overline{\phi}} = \phi$ for every *PIL* formula ϕ and $false = \overline{true}$. Then, the conjunction of two *PIL* formulas ϕ, ϕ' is defined by $\phi \wedge \phi' = \overline{(\overline{\phi} \vee \overline{\phi'})}$.

A *PIL* formula of the form $p_1 \wedge \ldots \wedge p_n$ where $n > 0$ and $p_i \in P$ or $\overline{p_i} \in P$ for every $1 \leq i \leq n$, is called a *monomial*. We shall simply denote a monomial $p_1 \wedge \ldots \wedge p_n$ by $p_1 \ldots p_n$.

Let ϕ be a *PIL* formula and a an interaction. We write $a \models_i \phi$ iff the formula ϕ evaluates to *true* by letting $p = true$ for every $p \in a$, and $p = false$ otherwise. It should be clear that $a \not\models_i false$ for every $a \in I(P)$. For every interaction a we define its characteristic monomial $m_a = \bigwedge_{p\in a} p \wedge \bigwedge_{p \notin a} \overline{p}$. Then, for every interaction a' we trivially get $a' \models_i m_a$ iff $a' = a$.

Throughout the paper P will denote a nonempty finite set of ports.

Definition 1. *The syntax of formulas of the* weighted *PIL over P and K is given by the grammar*

$$\varphi ::= k \mid \phi \mid \varphi \oplus \varphi \mid \varphi \otimes \varphi$$

where $k \in K$ and ϕ denotes a PIL formula.

We denote by $PIL(K, P)$ the set of all weighted *PIL* formulas φ over P and K. Next, we represent the semantics of formulas $\varphi \in PIL(K, P)$ as polynomials $\|\varphi\| \in K\langle I(P)\rangle$[1]. For the semantics of *PIL* formulas ϕ we use the satisfaction relation as defined above. In this way, we ensure that the semantics of *PIL* formulas ϕ gets only the values 0 and 1.

Definition 2. *Let $\varphi \in PIL(K, P)$. The semantics of φ is a polynomial $\|\varphi\| \in K\langle I(P)\rangle$. For every $a \in I(P)$ the value $\|\varphi\|(a)$ is defined inductively as follows:*

$$\|k\|(a) = k, \qquad \|\varphi \oplus \psi\|(a) = \|\varphi\|(a) \oplus \|\psi\|(a),$$

$$\|\phi\|(a) = \begin{cases} 1 & \text{if } a \models_i \phi \\ 0 & \text{otherwise} \end{cases}, \qquad \|\varphi \otimes \psi\|(a) = \|\varphi\|(a) \otimes \|\psi\|(a).$$

A polynomial $s \in K\langle I(P)\rangle$ is called *PIL-definable* if there is a formula $\varphi \in PIL(K, P)$ such that $s = \|\varphi\|$.

Remark 1. The reader should note that the semantics of the weighted *PIL* formulas $\phi \vee \phi$ and $\phi \oplus \phi$, where ϕ is a *PIL* formula, are different. Indeed assume that $a \in I(P)$ is such that $a \models_i \phi$. Then, by our definition above, we get $\|\phi \vee \phi\|(a) = 1$ whereas $\|\phi \oplus \phi\|(a) = 1 \oplus 1$.

Next we present an example of a weighted *PIL* formula.

Example 1. We recall from [11] the Master/Slave architecture for two masters M_1, M_2 and two slaves S_1, S_2 with ports m_1, m_2 and s_1, s_2, respectively. The monomial

$$\phi_{i,j} = s_i \wedge m_j \wedge \overline{s_{i'}} \wedge \overline{m_{j'}}$$

for every $1 \leq i, i', j, j' \leq 2$ with $i \neq i'$ and $j \neq j'$, defines the binary interaction between the ports s_i and m_j.

For every $1 \leq i, j \leq 2$ we consider the weighted *PIL* formula $\varphi_{i,j} = k_{i,j} \otimes \phi_{i,j}$ where $k_{i,j} \in K$. Hence, $k_{i,j}$ can be considered, according to the underlying semiring, as the "cost" for the implementation of the interaction $\phi_{i,j}$. For instance if K is the Viterbi semiring, then the value $k_{i,j} \in [0, 1]$ represents the probability of the implementation of the interaction between the ports s_i and m_j.

[1] Since P is finite, the domain of $\|\varphi\|$ is finite and in turn its support is also finite.

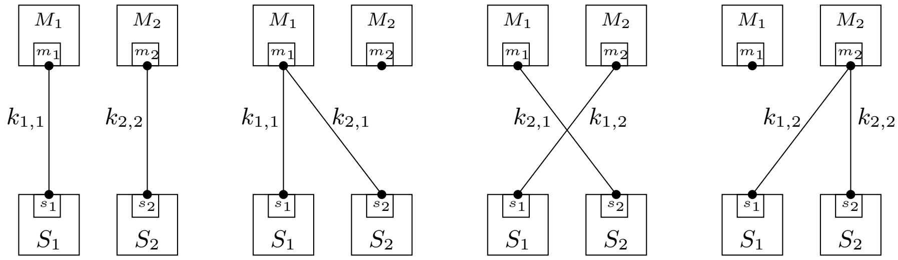

Fig. 1. Weighted Master/Slave architecture.

4 Weighted propositional configuration logic

In this section, we introduce and investigate the weighted propositional configuration logic. Firstly, we recall the propositional configuration logic of [11]. More precisely, the syntax of *propositional configuration logic* (*PCL* for short) formulas over P is given by the grammar

$$f ::= true \mid \phi \mid \neg f \mid f \sqcup f \mid f + f$$

where ϕ denotes a *PIL* formula. The operators $\neg$, $\sqcup$, and $+$ are called *complementation, union,* and *coalescing*, respectively. We define also the *intersection* $\sqcap$ and *implication* $\implies$ operators, respectively as follows: $f_1 \sqcap f_2 := \neg(\neg f_1 \sqcup \neg f_2)$, and $f_1 \implies f_2 := \neg f_1 \sqcup f_2$. To avoid any confusion, every *PCL* formula which is a *PIL* formula will be called an *interaction formula.* We let $C(P) = \mathcal{P}(I(P)) \setminus \{\emptyset\}$. For every *PCL* formula f and $\gamma \in C(P)$ we define the satisfaction relation $\gamma \models f$ inductively on the structure of f as follows:

$\gamma \models true$, $\quad$ $\gamma \models f_1 \sqcup f_2$ iff $\gamma \models f_1$ or $\gamma \models f_2$,

$\gamma \models \phi$ iff $a \models_i \phi$ for every $a \in \gamma$, $\quad$ $\gamma \models \neg f$ iff $\gamma \not\models f$,

$\gamma \models f_1 + f_2$ iff there exist $\gamma_1, \gamma_2 \in C(P)$ such that $\gamma = \gamma_1 \cup \gamma_2$, and $\gamma_1 \models f_1$ and $\gamma_2 \models f_2$.

The *closure* $\sim\! f$ of every *PCL* formula f, and the *disjunction* $f_1 \vee f_2$ of two *PCL* formulas f_1 and f_2 are defined, respectively by $\sim\! f := f + true$ and $f_1 \vee f_2 := f_1 \sqcup f_2 \sqcup (f_1 + f_2)$. Two *PCL* formulas f, f' are called *equivalent*, and we denote it by $f \equiv f'$, whenever $\gamma \models f$ iff $\gamma \models f'$ for every $\gamma \in C(P)$. We shall need the following lemma.

Lemma 1. *Let ϕ be a PIL formula. Then $\phi + \phi \equiv \phi$.*

Next we introduce our weighted *PCL*.

Definition 3. *The syntax of formulas of the* weighted *PCL over P and K is given by the grammar*

$$\zeta ::= k \mid f \mid \zeta \oplus \zeta \mid \zeta \otimes \zeta \mid \zeta \uplus \zeta$$

where $k \in K$, f denotes a PCL formula, and $\uplus$ denotes the coalescing *operator among weighted PCL formulas.*

Again, as for *PCL* formulas, to avoid any confusion, every weighted *PCL* formula which is a weighted *PIL* formula will be called a *weighted interaction formula*. We denote by $PCL(K, P)$ the set of all weighted *PCL* formulas over P and K. We represent the semantics of formulas $\zeta \in PCL(K, P)$ as polynomials $\|\zeta\| \in K\langle C(P)\rangle$. For the semantics of *PCL* formulas we use the satisfaction relation as defined previously.

Definition 4. *Let $\zeta \in PCL(K, P)$. The semantics of ζ is a polynomial $\|\zeta\| \in K\langle C(P)\rangle$. For every $\gamma \in C(P)$ the value $\|\zeta\|(\gamma)$ is defined inductively as follows:*

$\|k\|(\gamma) = k$, $\qquad \|\zeta_1 \oplus \zeta_2\|(\gamma) = \|\zeta_1\|(\gamma) \oplus \|\zeta_2\|(\gamma)$,

$\|f\|(\gamma) = \begin{cases} 1 & \text{if } \gamma \models f \\ 0 & \text{otherwise} \end{cases}$, $\qquad \|\zeta_1 \otimes \zeta_2\|(\gamma) = \|\zeta_1\|(\gamma) \otimes \|\zeta_2\|(\gamma)$,

$\|\zeta_1 \uplus \zeta_2\|(\gamma) = \bigoplus_{\gamma=\gamma_1\cup\gamma_2} (\|\zeta_1\|(\gamma_1) \otimes \|\zeta_2\|(\gamma_2))$.

Since the semantics of every weighted *PCL* formula is defined on $C(P)$, the sets γ_1 and γ_2 in $\|\zeta_1 \uplus \zeta_2\|(\gamma)$ are nonempty. A polynomial $s \in K\langle C(P)\rangle$ is called *PCL-definable* if there is a formula $\zeta \in PCL(K, P)$ such that $s = \|\zeta\|$. Two weighted *PCL* formulas ζ_1, ζ_2 are called equivalent, and we write $\zeta_1 \equiv \zeta_2$ whenever $\|\zeta_1\| = \|\zeta_2\|$.

The *closure* $\sim\zeta$ of every weighted *PCL* formula $\zeta \in PCL(K, P)$, and the *disjunction* $\zeta_1 \curlyvee \zeta_2$ of two weighted *PCL* formulas $\zeta_1, \zeta_2 \in PCL(K, P)$ are determined, respectively, by the following macros:

- $\sim\zeta := \zeta \uplus 1$,
- $\zeta_1 \curlyvee \zeta_2 := \zeta_1 \oplus \zeta_2 \oplus (\zeta_1 \uplus \zeta_2)$.

Trivially, $\|\sim\zeta\|(\gamma) = \bigoplus_{\gamma'\subseteq\gamma} \|\zeta\|(\gamma')$ for every $\gamma \in C(P)$.

For every *PCL* formula f over P and every weighted *PCL* formula $\zeta \in PCL(K, P)$, we consider also the macro:

- $f \implies \zeta := \neg f \oplus (f \otimes \zeta)$.

Then for $\gamma \in C(P)$, we get $\|f \implies \zeta\|(\gamma) = \|\zeta\|(\gamma)$ if $\gamma \models f$, and $\|f \implies \zeta\|(\gamma) = 1$ otherwise.

Example 2 (Example 1 continued). The four possible configurations of the Master/Slave architecture for two masters M_1, M_2 and two slaves S_1, S_2 with ports m_1, m_2 and s_1, s_2, respectively, are given by the *PIL* formula

$$(\phi_{1,1} \sqcup \phi_{1,2}) + (\phi_{2,1} \sqcup \phi_{2,2}).$$

We consider the weighted *PCL* formula

$$\zeta = \sim((\varphi_{1,1} \oplus \varphi_{1,2}) \uplus (\varphi_{2,1} \oplus \varphi_{2,2})).$$

Then for $\gamma \in C(\{m_1, m_2, s_1, s_2\})$ we get that $\|\zeta\|(\gamma)$ equals to

$$\bigoplus_{\gamma'\subseteq\gamma} \left(\bigoplus_{\gamma'=\gamma_1\cup\gamma_2} ((\|\varphi_{1,1}\|(\gamma_1) \oplus \|\varphi_{1,2}\|(\gamma_1)) \otimes (\|\varphi_{2,1}\|(\gamma_2) \oplus \|\varphi_{2,2}\|(\gamma_2))) \right).$$

Let us assume that $\gamma = \{\{s_1, m_1\}, \{s_1, m_2\}, \{s_2, m_1\}, \{s_2, m_2\}\}$. Then for $K = \mathbb{R}_{\min}$ the value $\|\zeta\|(\gamma)$ gets the form

$$\min_{\gamma' \subseteq \gamma} \left(\min_{\gamma' = \gamma_1 \cup \gamma_2} \left(\min\left(\|\varphi_{1,1}\|(\gamma_1), \|\varphi_{1,2}\|(\gamma_1)\right) + \min\left(\|\varphi_{2,1}\|(\gamma_2), \|\varphi_{2,2}\|(\gamma_2)\right)\right)\right)$$

which is the minimum "cost" of all the implementations of the Master/Slave architecture.

If $K = \mathbb{R}_{\max}$, then $\|\zeta\|(\gamma)$ equals to

$$\max_{\gamma' \subseteq \gamma} \left(\max_{\gamma' = \gamma_1 \cup \gamma_2} \left(\max\left(\|\varphi_{1,1}\|(\gamma_1), \|\varphi_{1,2}\|(\gamma_1)\right) + \max\left(\|\varphi_{2,1}\|(\gamma_2), \|\varphi_{2,2}\|(\gamma_2)\right)\right)\right)$$

which is the maximum "cost" of all the implementations of the Master/Slave architecture.

Finally assume K to be the Viterbi semiring. Then the value $k_{i,j}$ in $\varphi_{i,j}$ for every $1 \leq i, j \leq 2$, can be considered as the probability of the implementation of the interaction $\phi_{i,j}$. Hence, $\|\zeta\|(\gamma)$ equals to

$$\max_{\gamma' \subseteq \gamma} \left(\max_{\gamma' = \gamma_1 \cup \gamma_2} \left(\max\left(\|\varphi_{1,1}\|(\gamma_1), \|\varphi_{1,2}\|(\gamma_1)\right) \cdot \max\left(\|\varphi_{2,1}\|(\gamma_2), \|\varphi_{2,2}\|(\gamma_2)\right)\right)\right)$$

and represents the maximum probability of all the implementations of the Master/Slave architecture.

The next proposition summarizes several properties of our weighted *PCL* formulas.

Proposition 1. *Let* $\zeta_1, \zeta_2, \zeta_3 \in PCL(K, P)$. *Then*
(i) $(\zeta_1 \uplus \zeta_2) \uplus \zeta_3 \equiv \zeta_1 \uplus (\zeta_2 \uplus \zeta_3)$.
(ii) $\zeta_1 \uplus 0 \equiv 0$.
(iii) $\zeta_1 \uplus \zeta_2 \equiv \zeta_2 \uplus \zeta_1$.
(iv) $\zeta_1 \uplus (\zeta_2 \oplus \zeta_3) \equiv (\zeta_1 \uplus \zeta_2) \oplus (\zeta_1 \uplus \zeta_3)$.
(v) $\sim(\zeta_1 \oplus \zeta_2) \equiv \sim\zeta_1 \oplus \sim\zeta_2$.
(vi) $\sim(\zeta_1 \uplus \zeta_2) \equiv \sim\zeta_1 \otimes \sim\zeta_2$.
If in addition K is idempotent, then
(vii) $\sim(\zeta_1 \uplus \zeta_2) \equiv \sim\zeta_1 \uplus \sim\zeta_2$.
(viii) $\sim\sim\zeta_1 \equiv \sim\zeta_1$.
(ix) $\zeta_1 \curlyvee (\zeta_2 \oplus \zeta_3) \equiv (\zeta_1 \curlyvee \zeta_2) \oplus (\zeta_1 \curlyvee \zeta_3)$.

We aim to show that $\otimes$ does not distribute, in general, over $\uplus$. For this, we consider the semiring $(\mathbb{N}, +, \cdot, 0, 1)$ of natural numbers, the set of ports $P = \{p, q\}$ and the formulas $\zeta, \zeta_1, \zeta_2 \in PCL(\mathbb{N}, P)$ determined, respectively by $\zeta = 5 \oplus pq$, $\zeta_1 = pq \otimes 6$, and $\zeta_2 = pq \otimes 3$. We let $\gamma = \{\{p, q\}\}$ and by straightforward computations we get $\|\zeta \otimes (\zeta_1 \uplus \zeta_2)\|(\gamma) = 108$ and $\|(\zeta \otimes \zeta_1) \uplus (\zeta \otimes \zeta_2)\|(\gamma) = 648$. Hence $\zeta \otimes (\zeta_1 \uplus \zeta_2) \not\equiv (\zeta \otimes \zeta_1) \uplus (\zeta \otimes \zeta_2)$. Nevertheless, this is not the case whenever ζ is a *PIL* formula. More precisely, we state the subsequent proposition.

Proposition 2. *Let* ϕ *be a PIL formula over P and* $\zeta_1, \zeta_2 \in PCL(K, P)$. *Then*

$$\phi \otimes (\zeta_1 \uplus \zeta_2) \equiv (\phi \otimes \zeta_1) \uplus (\phi \otimes \zeta_2).$$

As it is already mentioned (cf. [11]), configuration logic has been developed as a fundamental platform to describe architecture styles. In the next example we show that weighted *PCL* in fact can formulate other types of problems.

Example 3. We consider the travelling salesman problem for 5 cities C_1, C_2, C_3, C_4, C_5, and assume C_1 to be the origin city. We aim to construct a weighted *PCL* formula, whose semantics computes the shortest distance of all the routes that visit every city exactly once and return to the origin city. We consider a port c_i for every city C_i $(1 \leq i \leq 5)$, hence $P = \{c_i \mid 1 \leq i \leq 5\}$. For every $1 \leq i, j, k, m, n \leq 5$ which are assumed to be pairwise disjoint, we define the monomials $\phi_{i,j}$ over P by

$$\phi_{i,j} = c_i c_j \overline{c}_k \overline{c}_m \overline{c}_n.$$

The interaction formulas $\phi_{i,j}$ represent the connection between the cities C_i and C_j. It should be clear that $\phi_{i,j} = \phi_{j,i}$ for every $1 \leq i \neq j \leq 5$. Assume that $K = \mathbb{R}_{\min}$ and for every $1 \leq i \neq j \leq 5$ we consider the weighted interaction formula

$$\varphi_{i,j} = k_{i,j} \otimes \phi_{i,j}$$

with $k_{i,j} \in \mathbb{R}_+$, where the values $k_{i,j}$ represent the distance between the cities C_i and C_j. Now we define the weighted *PCL* formula $\zeta \in PCL(\mathbb{R}_{\min}, P)$ as follows:

$$\zeta \equiv \sim \begin{pmatrix} (\varphi_{1,2} \uplus \varphi_{2,3} \uplus \varphi_{3,4} \uplus \varphi_{4,5} \uplus \varphi_{5,1}) \oplus (\varphi_{1,2} \uplus \varphi_{2,3} \uplus \varphi_{3,5} \uplus \varphi_{5,4} \uplus \varphi_{4,1}) \oplus \\ (\varphi_{1,2} \uplus \varphi_{2,4} \uplus \varphi_{4,5} \uplus \varphi_{5,3} \uplus \varphi_{3,1}) \oplus (\varphi_{1,2} \uplus \varphi_{2,5} \uplus \varphi_{5,4} \uplus \varphi_{4,3} \uplus \varphi_{3,1}) \oplus \\ (\varphi_{1,2} \uplus \varphi_{2,5} \uplus \varphi_{5,3} \uplus \varphi_{3,4} \uplus \varphi_{4,1}) \oplus (\varphi_{1,2} \uplus \varphi_{2,4} \uplus \varphi_{4,3} \uplus \varphi_{3,5} \uplus \varphi_{5,1}) \oplus \\ (\varphi_{1,3} \uplus \varphi_{3,2} \uplus \varphi_{2,4} \uplus \varphi_{4,5} \uplus \varphi_{5,1}) \oplus (\varphi_{1,3} \uplus \varphi_{3,2} \uplus \varphi_{2,5} \uplus \varphi_{5,4} \uplus \varphi_{4,1}) \oplus \\ (\varphi_{1,3} \uplus \varphi_{3,5} \uplus \varphi_{5,2} \uplus \varphi_{2,4} \uplus \varphi_{4,1}) \oplus (\varphi_{1,3} \uplus \varphi_{3,4} \uplus \varphi_{4,2} \uplus \varphi_{2,5} \uplus \varphi_{5,1}) \oplus \\ (\varphi_{1,4} \uplus \varphi_{4,2} \uplus \varphi_{2,3} \uplus \varphi_{3,5} \uplus \varphi_{5,1}) \oplus (\varphi_{1,4} \uplus \varphi_{4,3} \uplus \varphi_{3,2} \uplus \varphi_{2,5} \uplus \varphi_{5,1}) \end{pmatrix}.$$

Then for $\gamma = \{\{c_i, c_j\} \mid 1 \leq i \neq j \leq 5\}$, it is not difficult to see that the value $\|\zeta\|(\gamma)$ is the shortest distance of all the routes starting at C_1, visit every city exactly once, and return to C_1.

A weighted *PCL* formula can be constructed for the travelling salesman problem for any number n of cities. Indeed, assume the cities $C_1, \ldots, C_n$ with origin C_1. By preserving the above notations, we consider, for every $1 \leq i \neq j \leq n$, the interaction formula

$$\phi_{i,j} = c_i c_j \wedge \bigwedge_{k \in [n] \setminus \{i,j\}} \overline{c}_k$$

where $[n] = \{1, \ldots, n\}$, and the weighted interaction formula

$$\varphi_{i,j} = k_{i,j} \otimes \phi_{i,j}$$

with $k_{i,j} \in \mathbb{R}_+$, where the value $k_{i,j}$ represents the distance between the cities C_i and C_j. The required weighted *PCL* formula $\zeta \in PCL(\mathbb{R}_{\min}, P)$ is determined now as follows:

$$\zeta \equiv \sim \bigoplus_{\{i_1, \ldots, i_n\} \in \mathcal{CS}_n} \biguplus_{1 \leq j \leq n-1} \varphi_{i_j, i_{j+1}}$$

where $\mathcal{CS}_n$ denotes the set of all cyclic permutations of the first n positive integers such that clock-wise and anti-clock-wise cyclic permutations have been identified. It should be noted that $\mathrm{card}(\mathcal{CS}_n) = (n-1)!/2$. Then for $\gamma \in C(P)$ defined similarly as above, i.e., $\gamma = \{\{c_i, c_j\} \mid 1 \leq i \neq j \leq n\}$, the value $\|\zeta\|(\gamma)$ is the shortest distance of all the routes starting at C_1, visit every city exactly once, and return to C_1.

5 A full normal form for weighted *PCL* formulas

In the present section, we show that for every weighted *PCL* formula $\zeta \in PCL(K, P)$ we can effectively compute an equivalent formula of a special form. For this, we will use a corresponding result from [11]. More precisely, in that paper the authors proved that for every *PCL* formula f over P there exists a unique equivalent *PCL* formula of the form $\bigsqcup_{i \in I} \sum_{i \in J_i} m_{i,j}$ which is called *full normal form* (cf. Thm. 4.43. in [11]). The index sets I and J_i, for every $i \in I$, are finite and $m_{i,j}$'s are *full monomials*, i.e., monomials involving all ports from P. Hence, a full monomial is a monomial of the form $\bigwedge_{p \in P_+} p \wedge \bigwedge_{p \in P_-} \overline{p}$ where $P_+ \cup P_- = P$ and $P_+ \cap P_- = \emptyset$. We show that we can also effectively build a unique full normal form for every weighted *PCL* formula. Uniqueness is up to the equivalence relation. Then we will use this result to state that our weighted *PCL* is complete.

Definition 5. *A weighted PCL formula $\zeta \in PCL(K, P)$ is said to be in* full normal form *if there are finite index sets I and J_i for every $i \in I$, $k_i \in K$ for every $i \in I$, and full monomials $m_{i,j}$ for every $i \in I$ and $j \in J_i$ such that $\zeta = \bigoplus_{i \in I} \left(k_i \otimes \sum_{j \in J_i} m_{i,j}\right)$.*

By our definition above, for every full normal form we can construct an equivalent one satisfying the following statements:

i) $j \neq j'$ implies $m_{i,j} \not\equiv m_{i,j'}$ for every $i \in I$, $j, j' \in J_i$, and
ii) $i \neq i'$ implies $\sum_{j \in J_i} m_{i,j} \not\equiv \sum_{j \in J_{i'}} m_{i',j}$ for every $i, i' \in I$.

Indeed, for the first one, if $m_{i,j} \equiv m_{i,j'}$ for some $j \neq j'$, then since $m_{i,j}, m_{i,j'}$ are interaction formulas, by Lemma 1, we can replace the coalescing $m_{i,j} + m_{i,j'}$ with $m_{i,j}$. For (ii), let us assume that $\sum_{j \in J_i} m_{i,j} \equiv \sum_{j \in J_{i'}} m_{i',j}$ for some $i \neq i'$. Then, we can replace the sum $\left(k_i \otimes \sum_{j \in J_i} m_{i,j}\right) \oplus \left(k_{i'} \otimes \sum_{j \in J_{i'}} m_{i',j}\right)$ with the equivalent one $(k_i \oplus k_{i'}) \otimes \sum_{j \in J_i} m_{i,j}$. Hence, in the sequel, we assume that every full normal form satisfies Statements (i) and (ii).

We intend to show that for every weighted *PCL* formula $\zeta \in PCL(K, P)$ we can effectively construct an equivalent weighted *PCL* formula $\zeta' \in PCL(K, P)$ in full normal form. Moreover, ζ' will be unique up to the equivalence relation. We shall need a sequence of preliminary results. All index sets occurring in the sequel are finite.

Lemma 2. *Let $k_1, k_2 \in K$ and $\zeta_1, \zeta_2 \in PCL(K,P)$. Then*

$$(k_1 \otimes \zeta_1) \uplus (k_2 \otimes \zeta_2) \equiv (k_1 \otimes k_2) \otimes (\zeta_1 \uplus \zeta_2) .$$

Lemma 3. *Let J be an index set and m_j a full monomial for every $j \in J$. Then, there exists a unique $\overline{\gamma} \in C(P)$ such that for every $\gamma \in C(P)$ we have $\left\| \sum_{j \in J} m_j \right\| (\gamma) = 1$ if $\gamma = \overline{\gamma}$ and $\left\| \sum_{j \in J} m_i \right\| (\gamma) = 0$ otherwise.*

Proposition 3. *Let f be a PCL formula over P. Then there exist finite index sets I and J_i for every $i \in I$, and full monomials $m_{i,j}$ for every $i \in I$ and $j \in J_i$ such that*

$$f \equiv \bigoplus_{i \in I} \sum_{j \in J_i} m_{i,j} \equiv \bigoplus_{i \in I} \left(1 \otimes \sum_{j \in J_i} m_{i,j} \right) .$$

In particular

$$true \equiv \bigoplus_{\emptyset \neq N \subseteq M} \sum_{m \in N} m$$

where M is the set of all full monomials over P such that for every $m, m' \in M$, if $m \neq m'$, then $m \not\equiv m'$.

Lemma 4. *Let m_i, m'_j be full monomials for every $i \in I$ and $j \in J$. Then*

$$\left(\sum_{i \in I} m_i \right) \otimes \left(\sum_{j \in J} m'_j \right) \equiv \begin{cases} \sum_{i \in I} m_i & \text{if } \sum_{i \in I} m_i \equiv \sum_{j \in J} m'_j \\ 0 & \text{otherwise.} \end{cases}$$

Theorem 1. *Let K be a commutative semiring and P a set of ports. Then for every weighted PCL formula $\zeta \in PCL(K,P)$ we can effectively construct an equivalent weighted PCL formula $\zeta' \in PCL(K,P)$ in full normal form. Furthermore, ζ' is unique up to the equivalence relation.*

Proof (Sketch). We prove our theorem by induction on the structure of weighted *PCL* formulas ζ over P and K. Firstly, we show our claim for $\zeta = k$ with $k \in K$ and $\zeta = f$ where f is a *PCL* formula, using Proposition 3. Next, we consider weighted *PCL* formulas $\zeta_1, \zeta_2 \in PCL(K,P)$ and assume that $\zeta'_1 = \bigoplus_{i_1 \in I_1} \left(k_{i_1} \otimes \sum_{j_1 \in J_{i_1}} m_{i_1,j_1} \right)$, $\zeta'_2 = \bigoplus_{i_2 \in I_2} \left(k_{i_2} \otimes \sum_{j_2 \in J_{i_2}} m_{i_2,j_2} \right)$ are respectively their equivalent full normal forms. Then, we prove our claim for the case $\zeta = \zeta_1 \oplus \zeta_2$, $\zeta = \zeta_1 \otimes \zeta_2$, and $\zeta = \zeta_1 \uplus \zeta_2$ using Lemmas 2-4. Finally, it remains to show that ζ' is unique up to the equivalence relation. This is proved in a straightforward way using Statements (i) and (ii). □

A construction of the full normal form $\zeta' \in PCL(K,P)$ of every weighted *PCL* formula $\zeta \in PCL(K,P)$ can be done using our Theorem 1, and the Abstract Syntax Tree (AST)[2], in a similar way as it is done in [11]. More precisely, in our

[2] We refer the reader to [11] for the definition of the Abstract Syntax Tree.

case the leaves are labelled also by elements of the semiring K, and the nodes are labelled by additional symbols, namely the operators $\oplus$, $\otimes$, and $\uplus$. Whenever a node w of the AST is labelled by a symbol k, $\oplus$, $\otimes$, or $\uplus$, with $k \in K$, then every node of the path from the root to w is labelled by a symbol $\oplus$, $\otimes$, or $\uplus$.

Example 4 (Example 1 continued). We shall compute the full normal form of the weighted *PCL* formula

$$\zeta = \sim((\varphi_{1,1} \oplus \varphi_{1,2}) \uplus (\varphi_{2,1} \oplus \varphi_{2,2}))$$

which formalizes the weighted Master/Slave architecture for two masters M_1, M_2 and two slaves S_1, S_2 with ports m_1, m_2 and s_1, s_2, respectively. We have

$$\begin{aligned}
\zeta &= \sim((\varphi_{1,1} \oplus \varphi_{1,2}) \uplus (\varphi_{2,1} \oplus \varphi_{2,2})) \\
&\equiv (((k_{1,1} \otimes k_{2,1}) \otimes (\phi_{1,1} + \phi_{2,1})) \oplus ((k_{1,2} \otimes k_{2,1}) \otimes (\phi_{1,2} + \phi_{2,1})) \\
&\qquad \oplus ((k_{1,1} \otimes k_{2,2}) \otimes (\phi_{1,1} + \phi_{2,2})) \oplus ((k_{1,2} \otimes k_{2,2}) \otimes (\phi_{1,2} + \phi_{2,2}))) \uplus 1 \\
&\equiv \left(\bigoplus_{\emptyset \neq N \subseteq M} (k_{1,1} \otimes k_{2,1}) \otimes \left(\phi_{1,1} + \phi_{2,1} + \sum_{m \in N} m \right) \right) \\
&\qquad \oplus \left(\bigoplus_{\emptyset \neq N \subseteq M} (k_{1,2} \otimes k_{2,1}) \otimes \left(\phi_{1,2} + \phi_{2,1} + \sum_{m \in N} m \right) \right) \\
&\qquad \oplus \left(\bigoplus_{\emptyset \neq N \subseteq M} (k_{1,1} \otimes k_{2,2}) \otimes \left(\phi_{1,1} + \phi_{2,2} + \sum_{m \in N} m \right) \right) \\
&\qquad \oplus \left(\bigoplus_{\emptyset \neq N \subseteq M} (k_{1,2} \otimes k_{2,2}) \otimes \left(\phi_{1,2} + \phi_{2,2} + \sum_{m \in N} m \right) \right)
\end{aligned}$$

since $1 \equiv \bigoplus_{\emptyset \neq N \subseteq M} \left(1 \otimes \sum_{m \in N} m\right)$, where M is the set of all full monomials over P such that for every $m, m' \in M$, if $m \neq m'$, then $m \not\equiv m'$.

In the sequel, we intend to show that our weighted *PCL* is sound and complete. For this, we need firstly to introduce the notions of soundness and completeness for the weighted *PCL*. Let $\Sigma = \{\zeta_1, \ldots, \zeta_n\}$ be a set of weighted *PCL* formulas. Then we say that Σ *proves* the weighted *PCL* formula ζ and we write $\Sigma \vdash \zeta$ if ζ is derived by the formulas in Σ, using the axioms of *PCL* [11] and the equivalences of Propositions 1 and 2. Furthermore, we write $\Sigma \models \zeta$ if $\zeta_1 \equiv \ldots \equiv \zeta_n \equiv \zeta$.

Definition 6. *Let K be a commutative semiring and P a set of ports.*

(i) The weighted PCL over P and K is sound *if $\Sigma \vdash \zeta$ implies $\Sigma \models \zeta$ for every set of weighted PCL formulas Σ and weighted PCL formula ζ.*
(ii) The weighted PCL over P and K is complete *if $\Sigma \models \zeta$ implies $\Sigma \vdash \zeta$ for every set of weighted PCL formulas Σ and weighted PCL formula ζ.*

Theorem 2. *Let K be a commutative semiring and P a set of ports. Then the weighted PCL over P and K is sound and complete.*

6 Weighted first-order configuration logic

In this section, we equip our weighted *PCL* with first-order quantifiers and investigate the weighted first-order configuration logic. For this, we need to recall the first-order configuration logic from [11] for which, in addition, we prove several properties. We assume that $\mathcal{T} = \{T_1, \ldots, T_n\}$ is a finite set of component types such that instances of a component type have the same interface and behavior. We denote by C_T the set of all the components of type $T \in \mathcal{T}$, and we let $C_\mathcal{T} = \bigcup_{T \in \mathcal{T}} C_T$. A component c of type $T \in \mathcal{T}$ is denoted by $c : T$. The interface of every component type T has a distinct set of ports P_T. We set $P_\mathcal{T} = \bigcup_{T \in \mathcal{T}} P_T$. For every $B \subseteq C_\mathcal{T}$ we write P_B for the sets of ports of all the components in B. We denote by $c.p$ (resp. $c.P$) the port p (resp. the set of ports P) of component c. Furthermore, we assume that there is a universal component type U, such that every component or component set is of this type. Therefore, the set C_U is the set of all components of a model. Then, the syntax of *first-order configuration logic* (*FOCL* for short) formulas over $\mathcal{T}$ is given by the grammar

$$F ::= true \mid \phi \mid \neg F \mid F \sqcup F \mid F + F \mid \exists c : T(\Phi(c)).F \mid \textstyle\sum c : T(\Phi(c)).F$$

where ϕ denotes an interaction formula, c a component variable and $\Phi(c)$ a set-theoretic predicate on c. We omit Φ, in an *FOCL* formula, whenever $\Phi = true$.

Let $B \subseteq C_\mathcal{T}$ be a set of component instances of types from $\mathcal{T}$ and $\gamma \in C(P_B)$. Let also F be an *FOCL* formula without free variables (i.e., variables that are not in the scope of any quantifier). We define the satisfaction relation $(B, \gamma) \models F$ inductively on the structure of F as follows:

$(B, \gamma) \models true$, $\quad (B, \gamma) \models F_1 \sqcup F_2$ iff $(B, \gamma) \models F_1$ or $(B, \gamma) \models F_2$,

$(B, \gamma) \models \phi$ iff $\gamma \models \phi$, $\quad (B, \gamma) \models \neg F$ iff $(B, \gamma) \not\models F$,

$(B, \gamma) \models F_1 + F_2$ iff there exist $\gamma_1, \gamma_2 \in C(P_B)$ such that $\gamma = \gamma_1 \cup \gamma_2$, and $(B, \gamma_1) \models F_1$ and $(B, \gamma_2) \models F_2$,

$(B, \gamma) \models \exists c : T(\Phi(c)).F$ iff $(B, \gamma) \models \bigsqcup_{c' : T \in B \wedge \Phi(c')} F[c'/c]$,

$(B, \gamma) \models \sum c : T(\Phi(c)).F$ iff $\{c' : T \in B \mid \Phi(c')\} \neq \emptyset$ and $(B, \gamma) \models \sum_{c' : T \in B \wedge \Phi(c')} F[c'/c]$

where $F[c'/c]$ is obtained by F, by replacing all occurrences of c by c'. We let

- $\forall c : T(\Phi(c)).F := \neg \exists c : T(\Phi(c)).\neg F$.

The subsequent results refer to properties of the *FOCL* formulas.

Proposition 4. *Let F, F_1, F_2 be FOCL formulas. Then the following statements hold true.*

(i) $\sim\sim F = \sim F$.

(ii) $F \implies \sim F$.

(iii) $\neg\sim\neg F \implies F$.

(iv) $\sim(F_1 \sqcup F_2) \equiv \sim F_1 \sqcup \sim F_2$.

(v) $\sim(F_1 + F_2) \equiv \sim F_1 + \sim F_2$.

(vi) $\sim \exists c : T(\Phi(c)).F \equiv \exists c : T(\Phi(c)).(\sim F)$.

(vii) $\sim \sum c : T(\Phi(c)).F \equiv \sum c : T(\Phi(c)).(\sim F) \equiv \forall c : T(\Phi(c)).(\sim F)$.

(viii) $\exists c : T(\Phi(c)).(F_1 \sqcup F_2) \equiv \exists c : T(\Phi(c)).F_1 \sqcup \exists c : T(\Phi(c)).F_2$.

(ix) $\forall c : T(\Phi(c)).(F_1 \wedge F_2) \equiv \forall c : T(\Phi(c)).F_1 \wedge \forall c : T(\Phi(c)).F_2$.

(x) $\sum c : T(\Phi(c)).(F_1 + F_2) \equiv \sum c : T(\Phi(c)).F_1 + \sum c : T(\Phi(c)).F_2$.

(xi) $(\sim \sum c : T(\Phi(c)).F_1) \wedge (\sim \sum c : T(\Phi(c)).F_2) \equiv \forall c : T(\Phi(c)).(\sim(F_1 + F_2))$.

Proposition 5. *Let F_1, F_2 be two FOCL formulas over $\mathcal{T}$. Then*
(i) $\forall c : T(\Phi(c)).(F_1 + F_2) \implies \sum c : T(\Phi(c).F_1 + \sum c : T(\Phi(c).F_2$.
(ii) $\sum c : T(\Phi(c)).(F_1 \wedge F_2) \Rightarrow (\sum c : T(\Phi(c)).F_1) \wedge (\sum c : T(\Phi(c)).F_2)$.

The converse implications of both (i) and (ii) in Proposition 5 above do not in general hold.

Now we are ready to introduce the weighted *FOCL*.

Definition 7. *The syntax of formulas of the* weighted *FOCL over $\mathcal{T}$ and K is given by the grammar*

$$Z ::= k \mid F \mid Z \oplus Z \mid Z \otimes Z \mid Z \uplus Z \mid \bigoplus c : T(\Phi(c)).Z \mid \bigotimes c : T(\Phi(c)).Z \mid \biguplus c : T(\Phi(c)).Z$$

where $k \in K$ and F denotes an FOCL formula.

We denote by $FOCL(K, \mathcal{T})$ the class of all weighted *FOCL* formulas over $\mathcal{T}$ and K. We represent the semantics of formulas $Z \in FOCL(K, \mathcal{T})$ as polynomials $\|Z\| \in K \langle \mathcal{P}(C_{\mathcal{T}}) \times C(P_{\mathcal{T}}) \rangle$. For the semantics of *FOCL* formulas we use the satisfaction relation as defined previously.

Definition 8. *Let $Z \in FOCL(K, \mathcal{T})$. The semantics $\|Z\|$ is a polynomial in $K \langle \mathcal{P}(C_{\mathcal{T}}) \times C(P_{\mathcal{T}}) \rangle$. For every $B \in \mathcal{P}(C_{\mathcal{T}})$ and $\gamma \in C(P_{\mathcal{T}})$ we let $\|Z\|(B, \gamma) = 0$ if $\gamma \notin C(P_B)$. Otherwise, the value $\|Z\|(B, \gamma)$ is defined inductively as follows:*

$\|k\|(B, \gamma) = k$, $\quad \|Z_1 \oplus Z_2\|(B, \gamma) = \|Z_1\|(B, \gamma) \oplus \|Z_2\|(B, \gamma)$,

$\|F\|(B, \gamma) = \begin{cases} 1 & \text{if } (B, \gamma) \models F \\ 0 & \text{otherwise} \end{cases}$, $\quad \|Z_1 \otimes Z_2\|(B, \gamma) = \|Z_1\|(B, \gamma) \otimes \|Z_2\|(B, \gamma)$,

$\|Z_1 \uplus Z_2\|(B, \gamma) = \bigoplus_{\gamma = \gamma_1 \cup \gamma_2} (\|Z_1\|(B, \gamma_1) \otimes \|Z_2\|(B, \gamma_2))$,

$\|\bigoplus c : T(\Phi(c)).Z\|(B, \gamma) = \bigoplus_{c' : T \in B \wedge \Phi(c')} \|Z[c'/c]\|(B, \gamma)$,

$\|\bigotimes c : T(\Phi(c)).Z\|(B, \gamma) = \bigotimes_{c' : T \in B \wedge \Phi(c')} \|Z[c'/c]\|(B, \gamma)$,

$\|\biguplus c : T(\Phi(c)).Z\|(B, \gamma) = \bigoplus_{\gamma = \cup \gamma_{c'}, c' : T \in B \wedge \Phi(c')} \left(\bigotimes_{c' : T \in B \wedge \Phi(c')} \|Z[c'/c]\|(B, \gamma_{c'}) \right)$.

In the next proposition we establish the main properties of the weighted *FOCL* formulas.

Proposition 6. *Let $Z, Z_1, Z_2 \in FOCL(K, \mathcal{T})$. Then the following statements hold.*
(i) $\sim \bigoplus c : T(\Phi(c)).Z \equiv \bigoplus c : T(\Phi(c)).(\sim Z)$.
(ii) $\bigoplus c : T(\Phi(c)).(Z_1 \oplus Z_2) \equiv \bigoplus c : T(\Phi(c)).Z_1 \oplus \bigoplus c : T(\Phi(c)).Z_2$.
(iii) $\bigotimes c : T(\Phi(c)).(Z_1 \otimes Z_2) \equiv \bigotimes c : T(\Phi(c)).Z_1 \otimes \bigotimes c : T(\Phi(c)).Z_2$.
(iv) $\biguplus c : T(\Phi(c)).(Z_1 \uplus Z_2) \equiv (\biguplus c : T(\Phi(c)).Z_1) \uplus (\biguplus c : T(\Phi(c)).Z_2)$.

The subsequent examples constitute interesting applications of weighted *FOCL*. More precisely, in Example 5 we construct a weighted *FOCL* formula for the Master/Slave architecture for two Masters and three Slaves. In Example 2 we presented a weighted *PCL* formula for that architecture for two Masters and two Slaves. Nevertheless, that formula gets very complicated for several Masters and Slaves. On the contrary, the weighted *FOCL* formula of the next example can be easily modified for arbitrary numbers of Masters and Slaves and it is relatively simple. In Example 6 we built a formula for the Publish/Subscribe architecture style equipped with quantitative features.

Example 5 (Master/Slave architecture style). We intend to construct a weighted *FOCL* formula for two Masters and three Slaves. For this we need two types of components, namely M and S, for Masters and Slaves, respectively. Thus $\mathcal{T} = \{M, S\}$. We assume that every component of type M has only one port denoted by m and every component of type S has one port denoted by s, and we let $C_{\mathcal{T}} = \{b_1 : M, b_2 : M, d_1 : S, d_2 : S, d_3 : S\}$. We consider the weighted *FOCL* formula (with free variables c, c_1)

$$Z' = c.s \wedge c_1.m \otimes \left(\bigotimes_{\substack{i=1,2,3 \\ j=1,2}} ((c.s \equiv d_i.s \wedge c_1.m \equiv b_j.m) \implies k_{i,j}) \right) \otimes$$
$$\bigotimes c_2 : M(c_2 \neq c_1). \bigotimes c_3 : S(c_3 \neq c).(\overline{c_2.m} \wedge \overline{c_3.s})$$

and the weighted *FOCL* formula

$$Z = \sim\biguplus c : S. \left(\bigoplus c_1 : M.Z' \right).$$

Let $B = \{b_1 : M, b_2 : M, d_1 : S, d_2 : S, d_3 : S\}$ and $\gamma \in C(P_B)$. Then, by a straightforward computation, we can show that $\|Z\|(B, \gamma)$ equals to

$$\bigoplus_{\gamma' \subseteq \gamma} \left(\bigoplus_{\gamma' = \gamma_1 \cup \gamma_2 \cup \gamma_3} \left(\begin{pmatrix} k_{1,1} \otimes \left\| d_1.s \wedge b_1.m \wedge \overline{b_2.m} \wedge \overline{d_2.s} \wedge \overline{d_3.s} \right\| (\gamma_1) \oplus \\ k_{1,2} \otimes \left\| d_1.s \wedge b_2.m \wedge \overline{b_1.m} \wedge \overline{d_2.s} \wedge \overline{d_3.s} \right\| (\gamma_1) \end{pmatrix} \otimes \begin{pmatrix} k_{2,1} \otimes \left\| d_2.s \wedge b_1.m \wedge \overline{b_2.m} \wedge \overline{d_1.s} \wedge \overline{d_3.s} \right\| (\gamma_2) \oplus \\ k_{2,2} \otimes \left\| d_2.s \wedge b_2.m \wedge \overline{b_1.m} \wedge \overline{d_1.s} \wedge \overline{d_3.s} \right\| (\gamma_2) \end{pmatrix} \otimes \begin{pmatrix} k_{3,1} \otimes \left\| d_3.s \wedge b_1.m \wedge \overline{b_2.m} \wedge \overline{d_1.s} \wedge \overline{d_2.s} \right\| (\gamma_3) \oplus \\ k_{3,2} \otimes \left\| d_3.s \wedge b_2.m \wedge \overline{b_1.m} \wedge \overline{d_1.s} \wedge \overline{d_2.s} \right\| (\gamma_3) \end{pmatrix} \right) \right).$$

Assume for instance that $K = \mathbb{R}_{\max}$ and let $\gamma = \{\{d_1.s, b_1.m\}, \{d_1.s, b_2.m\}, \{d_2.s, b_1.m\}, \{d_2.s, b_2.m\}, \{d_3.s, b_1.m\}, \{d_3.s, b_2.m\}\}$. Then the semantics $\|Z\|(B, \gamma)$ firstly computes the weights of all the patterns that occur according to the set B, and finally returns the maximum of those weights. The weighted *FOCL* formula Z can be easily modified for any number of Masters and Slaves. Indeed, one has just to change accordingly the weighted formula Z'.

Example 6 (Publish/Subscribe architecture style). Publish/Subscribe is a software architecture, relating *publishers* who send messages and receivers, called *subscribers* (cf. for instance [4, 6]). The main characteristics of this architecture are as follows. The publishers characterize messages according to classes/topics but they do not know whether there is any subscriber who is interested in a concrete topic. Subscribers, on the other hand, express their interest in one or more topics and receive messages according to their interests in case such topics exist.

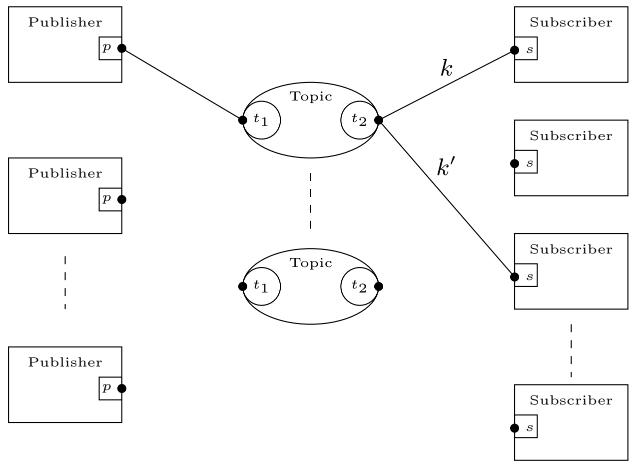

Fig. 2. Weighted Publish/Subscribe architecture.

There are three approaches to develop the Publish/Subscribe architecture, namely the *list-based*, the *broadcast-based*, and the *content-based*. Broadcast-based Publish/Subscribe and list-based Publish/Subscribe implementations can be broadly categorized as *topic-based* since they both use predefined subjects as many-to-many channels. More precisely, in a topic-based implementation, subscribers receive all messages published to the topics to which they subscribe, and all subscribers to a topic will receive the same messages. On the other hand, the publisher defines the topics of messages to which subscribers can subscribe. We intend to construct a weighted *FOCL* formula which formalizes the topic-based Publish/Subscribe architecture style. For this, we consider three types of components, the publishers, the topics and the subscribers denoted by the letters P, T, S, respectively. Hence, the set of component types is $\mathcal{T} = \{P, T, S\}$. The component P has one port p, T has two ports t_1 and t_2, and S has the port s. As it is mentioned above, the publishers do not have any knowledge of who and how many the subscribers are, and the same situation holds for the subscribers. In other words the publishers and the subscribers do not have any connection. Furthermore, a subscriber can receive a message from a topic, if at least one publisher has sent a message to that particular topic. The architecture is illustrated in Figure 2. Moreover, we should avoid interactions that transfer empty messages. The weights in our formula will represent the "priorities" that one subscriber gives to the topics. Next, we describe the required weighted *FOCL* formula for the Publish/Subscribe architecture. Assume that we have a component of type P namely $c_3 : P$ and a component $c_2 : T$ of type T. If the publisher $c_3 : P$ will send a message to that topic $c_2 : T$, then this interaction is represented by the formula $c_3.p \wedge c_2.t_1$. However, we must ensure that no other components of type P, type T, or type S will interact. This case is obtained by the formula Z_1 below:

$$Z_1 = \forall d_1 : P(d_1 \neq c_3). \left(\begin{array}{l} \forall d_2 : T(d_2 \neq c_2). \\ \qquad (\forall d_3 : S. \left(\overline{d_1.p} \wedge \overline{d_2.t_1} \wedge \overline{d_2.t_2} \wedge \overline{d_3.s} \wedge \overline{c_2.t_2}\right)) \end{array} \right).$$

Then the *FOCL* formula

$$Z_2 = \sim (c_3.p \wedge c_2.t_1 \wedge Z_1)$$

characterizes interactions between a publisher and a topic. Assume now that a message has been sent to the component $c_2 : T$. Then this message can be sent to a subscriber $c_1 : S$ who has expressed interest in the same topic. This interaction is represented by the *FOCL* formula $c_2.t_2 \wedge c_1.s$. Similarly, as in the previous case, in this interaction there must participate not any other subscribers, topics, or publishers. This case is implemented by the formula

$$Z_3 = \forall d_1 : P. \left(\forall d_2 : T(d_2 \neq c_2). \left(\begin{array}{l} \forall d_3 : S(d_3 \neq c_1). \\ \qquad \left(\overline{d_1.p} \wedge \overline{d_2.t_1} \wedge \overline{d_2.t_2} \wedge \overline{d_3.s} \wedge \overline{c_2.t_1}\right) \end{array} \right) \right),$$

and thus we get

$$\sim (c_2.t_2 \wedge c_1.s \wedge Z_3).$$

However, the formula that characterizes an interaction between a topic and a subscriber is not yet complete. As it is mentioned above, each subscriber gives a certain priority to every topic that is interested in. So, in the last formula above we have also to "add" the corresponding weights. Therefore, we derive the weighted *FOCL* formula Z_4 containing the priorities of two subscribers $s_1 : S$, $s_2 : S$ to the topics $r_1 : T$, $r_2 : T$, and $r_3 : T$ as follows:[3]

$$Z_4 = \bigotimes_{\substack{i=1,2,3 \\ j=1,2}} \left((c_2.t_2 \equiv r_i.t_2 \wedge c_1.s \equiv s_j.s) \implies k_{i,j}\right).$$

We conclude to the following weighted *FOCL* formula Z_5 which characterizes an interaction between a subscriber and a topic with its corresponding weight

$$Z_5 = (\sim (c_2.t_2 \wedge c_1.s \wedge Z_3)) \otimes Z_4.$$

Finally, in order to complete the formula that formalizes the Publish/Subscribe architecture style, we have to generalize the above procedure for every subscriber. Indeed, the required formula must check for every subscriber whether there exists a topic that the subscriber is interested in, and also if there exists a publisher that has sent a message to that topic, so that the subscriber can receive it. Therefore, we define the weighted *FOCL* formula

$$Z = \bigotimes c_1 : S. \left(\bigoplus c_2 : T. \left(\bigoplus c_3 : P. (Z_2 \uplus Z_5) \right) \right)$$

[3] For simplicity we consider concrete numbers of subscribers and topics. Trivially, one can modify the weighted *FOCL* formula Z_4 for arbitrarily many subscribers and topics.

which characterizes the Publish/Subscribe architecture style. Clearly Z can describe the Publish/Subscribe architecture for any number of subscribers by modifying accordingly the weighted *FOCL* formula Z_4.

Assume, for instance, that $C_{\mathcal{T}} = \{p_1 : P, p_2 : P, p_3 : P, p_4 : P, r_1 : T, r_2 : T, r_3 : T, s_1 : S, s_2 : S\}$ is the set of all the components, and K is the Viterbi semiring. Let also $B = \{p_1 : P, p_2 : P, r_1 : T, r_2 : T, r_3 : T, s_1 : S, s_2 : S\} \subseteq C_{\mathcal{T}}$. Then for every $\gamma \in C(P_B)$ we get

$$
\begin{aligned}
&\|Z\|(B,\gamma)\\
&= \left\| \bigotimes c_1 : S. \left(\bigoplus c_2 : T. \left(\bigoplus c_3 : P. \left(Z_2 \uplus Z_5 \right) \right) \right) \right\| (B,\gamma)\\
&= \prod_{c_1':S\in B} \left\| \bigoplus c_2 : T. \left(\bigoplus c_3 : P. (Z_2 \uplus Z_5)[c_1'/c_1] \right) \right\| (B,\gamma)\\
&= \prod_{c_1':S\in B} \left(\max_{c_2':T\in B} \left(\left\| \bigoplus c_3 : P. (Z_2 \uplus Z_5)[c_1'/c_1, c_2'/c_2] \right\| \right) \right) (B,\gamma)\\
&= \prod_{c_1':S\in B} \left(\max_{c_2':T\in B} \left(\max_{c_3':P\in B} \left(\left\| (Z_2 \uplus Z_5)[c_1'/c_1, c_2'/c_2, c_3'/c_3] \right\| (B,\gamma) \right) \right) \right)\\
&= \left(\max \begin{pmatrix} \max \begin{pmatrix} \|(Z_2 \uplus Z_5)[s_1/c_1, r_1/c_2, p_1/c_3]\| (B,\gamma), \\ \|(Z_2 \uplus Z_5)[s_1/c_1, r_1/c_2, p_2/c_3]\| (B,\gamma) \end{pmatrix}, \\ \max \begin{pmatrix} \|(Z_2 \uplus Z_5)[s_1/c_1, r_2/c_2, p_1/c_3]\| (B,\gamma), \\ \|(Z_2 \uplus Z_5)[s_1/c_1, r_2/c_2, p_2/c_3]\| (B,\gamma) \end{pmatrix}, \\ \max \begin{pmatrix} \|(Z_2 \uplus Z_5)[s_1/c_1, r_3/c_2, p_1/c_3]\| (B,\gamma), \\ \|(Z_2 \uplus Z_5)[s_1/c_1, r_3/c_2, p_2/c_3]\| (B,\gamma) \end{pmatrix} \end{pmatrix} \right) \cdot \\
&\qquad \cdot \left(\max \begin{pmatrix} \max \begin{pmatrix} \|(Z_2 \uplus Z_5)[s_2/c_1, r_1/c_2, p_1/c_3]\| (B,\gamma), \\ \|(Z_2 \uplus Z_5)[s_2/c_1, r_1/c_2, p_2/c_3]\| (B,\gamma) \end{pmatrix}, \\ \max \begin{pmatrix} \|(Z_2 \uplus Z_5)[s_2/c_1, r_2/c_2, p_1/c_3]\| (B,\gamma), \\ \|(Z_2 \uplus Z_5)[s_2/c_1, r_2/c_2, p_2/c_3]\| (B,\gamma) \end{pmatrix}, \\ \max \begin{pmatrix} \|(Z_2 \uplus Z_5)[s_2/c_1, r_3/c_2, p_1/c_3]\| (B,\gamma), \\ \|(Z_2 \uplus Z_5)[s_2/c_1, r_3/c_2, p_2/c_3]\| (B,\gamma) \end{pmatrix} \end{pmatrix} \right).
\end{aligned}
$$

Let $\gamma = \{\{p_1.p, r_1.t_1\}, \{p_1.p, r_3.t_1\}, \{p_2.p, r_1.t_1\}, \{r_1.t_2, s_1.s\}, \{r_2.t_2, s_2.s\}, \{r_3.t_2, s_2.s\}\}$. By straightforward computations we get $\|Z\|(B,\gamma) = k_{1,1} \cdot k_{3,2}$, which represents the greatest combination of priorities of the subscribers according to γ.

Conclusion

We introduced a weighted *PCL* over a commutative semiring K and investigated several properties of the class of polynomials obtained as semantics of this logic. For some of that properties we required our semiring to be idempotent. We proved that for every weighted *PCL* formula ζ we can effectively construct an equivalent one ζ' in full normal form. Furthermore, ζ' is unique up to the equivalence relation. This result implied that our logic is complete, and we showed that it is also sound. Weighted *PCL* describes nicely, architectures with quantitative characteristics. We extended the weighted *PCL* to weighted first-order configuration logic with which we could represent architecture styles equipped with quantitative features. We proved several properties for the class of polynomials definable by weighted first-order configuration logic. We also provided examples of weighted architecture styles. In our future work we will study decidability results and weighted second-order configuration logic. It is an open problem whether we can develop the theory of this paper by relaxing the commutativity property of the semiring K and thus obtaining our results for a larger class of semirings. Furthermore, it should be very interesting to investigate the weighted *PCL* and its first-order extension over more general weight structures which can describe further properties like average, limit inferior, limit superior, and discounting (cf. for instance [2]).

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